ABSTRACT
The main purpose of this paper is to apply fuzzy set theory to the leasing credit granting management. A new leasing credit granting model is proposed to improve the quality of leasing credit granting decision-making under fuzzy environment. A fuzzy breakeven analysis based on the cost-volume-profit analysis by using the concepts of triangular fuzzy numbers and linguistic variables is discussed. A leasing client's granting quality determination model is constructed by combining fuzzy breakeven analysis with Markov chain. An algorithm of the proposed model is developed to see if there is any improvement in credit quality of client after leasing by a decision rule defined in this paper.

Keywords: Leasing; Credit Granting Decision; Cost-Volume-Profit Analysis; Fuzzy Number; Linguistic Variable; Markov Chain.

1. INTRODUCTION
There is a considerable amount of literature discussing the factors that influence a corporate leasing policy and rents[7,9,12,13]. During the leasing credit granting process, leasing corporate officer can evaluate the client's credit by taking into account both the risk coverage and profit coverage two sides of clients. Bank credit granting problem has been investigated extensively on the risk coverage[1,2,6,10,11]. Most of them used different sorts of multivariate statistical methods to find out appropriate client's attributes, which can distinguish good credit from bad credit of the clients. While some aspects of leasing credit granting decision-making and the quality of credit after leasing are not explored yet. Although Wu et al.[15] combined the Markov chain with cost-volume-profit analysis to the bank credit granting problem, they didn't consider the fuzzy nature of the problem. The profit side and cost-volume-profit analysis are considered in this paper for making a leasing credit granting decision.

When the leasing officer uses cost-volume-profit analysis as a tool to make the credit granting decision, he evaluates the client's credit according to its past financial statements and operating situation. But in this evaluation process, the officer has to consider some certain variables which "impreciseness" or "fuzziness" is involved. So if such fuzziness is not taken into account in cost-volume-profit analysis, the officer may make the erroneous decision and undertake much potential risk [3]. This paper is intended to resolve this problem and compute the fuzzy breakeven point based on fuzzy numbers, linguistic variables, and cost-volume-profit analysis.

Furthermore, the credit granting quality of
the lessee is another important aspect that must be considered when the officer estimates his credit. Because the lessee's credit quality will influence the lessor's decision, this paper is also attempted to evaluate a lessee's credit quality, by comparing the operating performance before and after leasing, for each customer to see if there's any improvement through Markov chain.

With these motivations, the purposes of this paper is two-fold:
(1) to construct a fuzzy breakeven analysis for leasing credit evaluation by using fuzzy set theory to deal with the uncertainty of variables in cost-volume-profit analysis;
(2) to provide an algorithm combining by Markov chain and fuzzy breakeven analysis for monitoring client's credit granting quality.

Section 2 briefly discusses the main concepts and methods used in this paper. An algorithm is proposed in Section 3 for leasing credit granting decision-making. Section 4 and Section 5 deal with an example and conclusion, respectively.

2. PRELIMINARIES

In this section the basic concepts and methods used in this paper are briefly discussed, including cost-volume-profit analysis, fuzzy number, linguistic variable, and Markov chain.

2.1. Cost-Volume-Profit (CVP) Analysis

Suppose that the main purpose of a corporate operating is to earn profit in the short-run. Then, the profit becomes an index to evaluate a company’s performance. There are many factors could affect a company’s performance, including the internal and external factors. The external factors are political, economic, cultural, technological, and so on. The internal factors are business functions including marketing, production, human resources, research and development, accounting and finance. The CVP model can be divided into four variables: (1) price, (2) volume, (3) fixed cost and (4) variable cost. The CVP model is defined as follows:

\[ Z = Q \times (P-V) - FC \]  \hspace{1cm} (1)

Here, \( Z \) : profit, \( Q \) : sales volume in units, \( P \) : unit price, \( FC \) : total fixed cost, \( V \) : average variable cost.

According to CVP model, when the company’s profit is zero, the sales volume in units \( Q \) and sales of breakeven point \( (S=P \times Q) \) can be derived from the following equations :

\[
\begin{align*}
0 &= Q \times (P-V) - FC \\
Q &\times (P-V) = FC \\
Q &= FC/(P-V) \\
S &= FC/(1-V/P) 
\end{align*}
\]  \hspace{1cm} (2)

In this paper, the total fixed cost(FC) is composed by the wage (FC1) and the fixed expenditure of manufacture (FC2). The total variable cost (VC) is composed by the material cost(VC1), expenditure of variable manufacture(VC2) and direct labor cost(VC3). A common technique for identifying profitable prices is breakeven analysis, which involves determining the sales volume needed to cover all costs at a specific price. The level of sales (S) at which total revenues \((P \times Q)\) equal total costs \((FC+VC)\).

If the sales is higher than the breakeven point \( S \), profit occurs. Conversely, if the sales is lower than the breakeven point \( S \), the company has loss.

However, according to Dickinson[4], Chang
and Yuan[3] researches, the variables of CVP analysis should be estimated in an uncertain environment. These variables are fuzzy in nature. In this paper we adopt the fuzzy CVP analysis based on fuzzy numbers and linguistic variables to determine the leasing credit granting decision-making.

2.2 Fuzzy Set Theory

Fuzzy set theory was introduced by Zadeh[16] to deal with the problems in which the fuzzy phenomena is presented. In this section, the notations and concepts of the fuzzy set theory will be briefly introduced.

2.2.1 Fuzzy Set

Let X be a collection of objects, called the universe with the element is denoted by x. A fuzzy set A in X is a set of ordered pairs: A = \( \{ (x, \mu_A(x)) | x \in X \} \), where \( \mu_A(x) \) is the membership function of x in A which associates with each element x in X a real number in the interval [0,1]. The larger \( \mu_A(x) \), the stronger the degree of belongingness for x in A.

2.2.2 Fuzzy Number

The fuzzy number[5] is very useful in promoting the representation and information processing under fuzzy environment. A fuzzy number A in R (real line) is a triangular fuzzy number, if its membership function \( \mu_A(x) : R \rightarrow [0,1] \) is equal to

\[
\mu_A(x) = \begin{cases} 
\frac{(x-c)}{(a-c)}, & c \leq x \leq a, \\
\frac{(x-b)}{(a-b)}, & a \leq x \leq b, \\
0, & \text{otherwise},
\end{cases}
\]

where, \( c < a < b \). The triangular fuzzy number can be denoted by \( A = (c, a, b) \). The parameter "a" gives the maximal grade of \( \mu_A(a) = 1 \). The triangular fuzzy number used in this paper is a normal and convex fuzzy set of R, i.e., \( \mu_A(a) = 1 \), it is the most possible value of the evaluation data. The "c" and "b" are the lower and upper bounds of the area of the evaluation data, which are used to reflect the fuzziness of the data. The reason of using the triangular fuzzy number is due to that it is intuitively easy for the decision-makers to perform evaluation. The non-fuzzy number "a" can be expressed as (a, a, a).

2.2.3 The Arithmetic Operations on Fuzzy Numbers

Zadeh introduced the extension principle[16] to find the membership function after mapping fuzzy sets through a function. By extension principle, we can define the arithmetic operations on triangular fuzzy number as follows.

Let \( A_1 = (c_1, a_1, b_1) \) and \( A_2 = (c_2, a_2, b_2) \). We have

1. fuzzy addition,(+):
\[
A_1(+)A_2 = (c_1 + c_2, a_1 + a_2, b_1 + b_2).
\]

2. fuzzy subtraction,(-):
\[
A_1(-)A_2 = (c_1 - b_2, a_1 - a_2, b_1 - c_2).
\]

3. fuzzy multiplication,(x):
\[
A_1(x)A_2 \equiv (c_1c_2, a_1a_2, b_1b_2), \quad c_1, c_2 > 0;
\]

4. fuzzy division,(÷):
\[
A_1(÷)A_2 \equiv (c_1/b_2, a_1/a_2, b_1/c_2), \quad c_1, c_2 > 0.
\]

2.2.4 Linguistic Variable

By a linguistic variable[17] we mean a variable whose values are words or sentences in a natural or artificial language. A linguistic variable is characterized by a quintuple \( (\chi, T(\chi), U, G, M) \) in which \( \chi \) is the name of the variable; \( T(\chi) \) is the term-set of \( \chi \), that is, the set of its linguistic values; \( U \) is a universe of discourse; \( G \) is a syntactic rule which
generates the term in $T(\chi)$; and $M$ is a semantic rule which associates with each linguistic value $X$ its meaning, $M(X)$, where $M(X)$ denotes a fuzzy subset of $U$. The concept of a linguistic variable provides a means of approximate characterization of phenomena which are too complex or too ill-defined to be amenable to description in conventional quantitative terms.

In a linguistic variable, linguistic terms representing approximate values of a base variable, the values of which are real numbers within a specific range, are captured by appropriate fuzzy numbers. Each of the basic linguistic terms described in this paper (very low, low, medium, high, very high) is assigned to one of five triangular fuzzy numbers by a semantic rule, as shown in Figure 1. The fuzzy numbers, whose membership functions have the triangular shapes, are defined on the interval $[0,1]$, the range of the base variable.

2.2.5 Ranking of Fuzzy Number

In many fuzzy decision problems, the final scores of alternatives are represented in terms of fuzzy numbers. In order to express a crisp preference of alternatives, we need a method for constructing a crisp total ordering from fuzzy numbers. Numerous methods for total ordering of fuzzy numbers have been suggested in the literature. Each method appears to have some advantages as well as disadvantages.

In this paper, Kim and Park's method[8] is used to compute the ranking values of fuzzy numbers.

Define the maximizing set

$M = \{ (x, \mu_M(x)) | x \in R \}$

with

$$\mu_M(x) = \begin{cases} \frac{(x - x_1)}{(x_2 - x_1)}, & x_1 \leq x \leq x_2, \\ 0, & \text{otherwise}, \end{cases}$$

and minimizing set $G = \{ (x, \mu_G(x)) | x \in R \}$

with

$$\mu_G(x) = \begin{cases} \frac{(x - x_1)}{(x_2 - x_1)}, & x_1 \leq x \leq x_2, \\ 0, & \text{otherwise}, \end{cases}$$

where $x_1 = \inf S, x_2 = \sup S, S = \bigcup_{i=1}^{m} F_i$,

$F_i = \{ x | \mu_{F_i}(x) > 0 \}, \quad i = 1, 2, ..., m$. Define the optimistic utility $U_M(F_i)$ and pessimistic utility $U_G(F_i)$ of each fuzzy number as

$U_M(F_i) = \sup_x (\mu_{F_i}(x) \land \mu_M(x))$,

and

$U_G(F_i) = 1 - \sup_x (\mu_{F_i}(x) \land \mu_G(x))$,

for $i = 1, 2, ..., m$, where $\land$ means min.

Define ranking value $U_\beta(F_i)$ of fuzzy number as

$$U_\beta(F_i) = \beta U_M(F_i) + (1 - \beta) U_G(F_i), \quad 0 \leq \beta \leq 1.$$
attitude. If $\beta > 0.5$, it implies that the decision maker is a risk lover. If $\beta < 0.5$, the decision maker is a risk averter. In this paper, $\beta = 0.5$ is applied, i.e., the attitude of decision maker is neutral to the risk. The ranking values $U_T(F_i)$ can be approximately obtained by

$$U_T(F_i) = \left( (Z_i - x_i) / (x_2 - x_1 - Q_i + Z_i) \right) + 1 - \left( (x_2 - Y_i) / (x_2 - x_1 + Q_i - Y_i) \right) / 2$$

for $i = 1, 2, ..., m$, where $x_i = \min\{Y_i, Y_2, ..., Y_m\}$, $x_2 = \max\{Z_1, Z_2, ..., Z_m\}$, and $F_i = (Y_i, Q_i, Z_i)$.

2.3 Markov Chain

Markovian property means that a random dynamic system that if its present state is known, then its past state has no influence on the future state. Dynamic programming determines the optimum solution to an $n$-variable problem by decomposing it into $n$ stages with each stage comprising a single variable subproblem. Markovian dynamic programming is a process of probability. It presents an application of dynamic programming to the situation of a stochastic decision process that can be described by a finite number of states. The transition probabilities between the states are described by a Markov chain. The reward structure of the process is also described by a matrix whose individual elements represent the revenue (or cost) resulting from moving from one state to another. Both the transition and revenue matrices depend on the decision alternatives available to the decision-maker. The objective of the problem is to determine the optimal policy that maximizes the expected revenue over a finite or infinite number of states [14].

In this paper the transition matrix in the Markov chain is used to predict lessees’

granting credit after the lease. Some notations are defined as follow. Let

$P^1$: the transition probabilities before leasing

$P^2$: the transition probabilities after leasing

$p^1_{ij}$: before leasing, the probability that if the corporate is $i^{th}$ state on the first stage, then it moves to $j^{th}$ state on the second stage

$p^2_{ij}$: after leasing, the probability that if the corporate is $i^{th}$ state on the first stage, then it moves to $j^{th}$ state on the second stage

$$P^1 = \begin{bmatrix} P^1_{11} & P^1_{12} & P^1_{13} \\ P^1_{21} & P^1_{22} & P^1_{23} \\ P^1_{31} & P^1_{32} & P^1_{33} \end{bmatrix}$$

$$P^2 = \begin{bmatrix} P^2_{11} & P^2_{12} & P^2_{13} \\ P^2_{21} & P^2_{22} & P^2_{23} \\ P^2_{31} & P^2_{32} & P^2_{33} \end{bmatrix}$$

$i = j = 1$: the breakeven state is “good”
i = j = 2: the breakeven state is “fair”
i = j = 3: the breakeven state is “poor”

$$R^1 = ||r^1 - S_A||:$$ the reward matrix before leasing

$$R^2 = ||r^2 - S_A||:$$ the reward matrix after leasing

$r^k_{ij} (i = j = 1, 2, 3, k = 1, 2):$ the predictive sales

$r^1_{ij} - S_A$: before leasing, the excess operating income results from reaching state $j$ at next year from state $i$ at the current year

$r^2_{ij} - S_A$: after leasing, the excess operating income resulting from reaching state $j$ at next year from state $i$ at the current year
Sb : sales of the breakeven interval before leasing
Sa : sales of the breakeven interval after leasing

Now we suppose there are s kinds of states in every stage (every year). The symbol i is the state of the system current year and the symbol j is the state of the system next year. Here k represents the decision of action available to the leasing officer. \( p_k^i \) and \( r_j \) means the probability and reward that if the corporate is \( i^{th} \) state on the first stage and chooses the decision \( k \), then it moves to \( j^{th} \) state on the second stage. To a leasing corporate, s is defined to be equal to 3, that is \{good, fair, poor\}. The symbol \( f_{n}(i) \) is the optimal expected revenue of stages \( n,n+1,...,N \) given that the state of the system at the beginning of year \( n \) is \( i \). The backward recursive equation relating \( f_{n} \) and \( f_{n+1} \) can be expressed as

\[
f_n(i) = \max\left\{ \sum_{j=1}^{s} p_k^i [r_j + f_{n+1}(j)] \right\},
\]

\( n=1,2,...,N-1 \),

where \( f_{N+1}(j) \equiv 0 \) for all \( j \). (4)

A justification for the equation is the cumulative excess operating income \( r_j + f_{n+1}(j) \), resulting from reaching state \( j \) at stage \( n+1 \) from state \( i \) at stage \( n \) occurs with probability \( p_k^i \). Letting

\[
u_k^i = \sum_{j=1}^{s} p_k^i r_j ,
\]

Where \( u_k^i \) means if the leasing officer chooses the decision \( k \), then the excess operating income which is resulting from reaching state \( j \) at stage \( n+1 \) from state \( i \) at stage \( n \) occurs with probability \( p_k^i \).

3. AN ALGORITHM FOR CREDIT GRANTING AND CREDIT QUALITY

In this section, fuzzy set theory is applied to client's credit granting decision-making in leasing under fuzzy environment. A fuzzy breakeven analysis for credit granting decision-making based on the cost-volume-profit analysis, fuzzy numbers, and linguistic variables is discussed. Then we combine the fuzzy breakeven analysis with the transition matrix in the Markov chain to determine the quality of leasing credit granting to see if there is any improvement of clients after leasing.

An algorithm for the leasing officer to evaluate its client's credit granting quality described as follows.

Step 1: Use the sales, volume, fixed and variable cost which are measured by the questionnaire and Eq.(2) to compute \( S_b \). Here, \( S_b \) is a fuzzy number.

Step 2: Calculate \( S_a \).
Let \( \Delta A_1 = (c_1, a_1, b_1) \) be incremental fuzzy number of the breakeven interval after leasing that is from the expected values of the variables of Step 1 by using linguistic values.

\[
S_a \equiv S_b (\times ) (1(+ \Delta A_1 ))
\]

(6)

From Step 1-2, we can determine the leasing credit granting decision-making by comparing \( S_b \) with \( S_a \) through the ranking values of \( S_b \) and \( S_a \).

Step 3: From the questionnaire, we can obtain the client's reward matrix before leasing \( R^1 \) and after leasing \( R^2 \).
\[
\begin{bmatrix}
\begin{array}{ccc}
    r'_{11} & r'_{12} & r'_{13} \\
    r'_{21} & r'_{22} & r'_{23} \\
    r'_{31} & r'_{32} & r'_{33}
\end{array}
\end{bmatrix}
\]
\[R^1=
\begin{bmatrix}
\begin{array}{ccc}
    r^2_{11} & r^2_{12} & r^2_{13} \\
    r^2_{21} & r^2_{22} & r^2_{23} \\
    r^2_{31} & r^2_{32} & r^2_{33}
\end{array}
\end{bmatrix}
\]

Step 4: Determine the transition probabilities \( P^1 \) and \( P^2 \) through leasing officer by questionnaire.

Step 5: By taking 1-year period, use Eq.(5), \( P^3 \), and \( R^2 \) to calculate the excess operating income \( u^k \).

Step 6: Determine the quality of leasing credit granting.

From Step 1 to Step 5, we can obtain every client’s excess operating income interval before and after leasing and use Eq.(3) to calculate the ranking values of the client’s excess operating income before and after leasing. Let the fuzzy numbers of the client’s excess operating income before and after leasing are \((e_b, d_b, f_b)\) and \((e_a, d_a, f_a)\), respectively, and let \(U (\ast)\) be the ranking value of fuzzy number \(\ast\). Then we can use the following rules to determine whether the quality of leasing credit granting is improved or not.

Rule 1: If \( U((e_b, d_b, f_b)) > U((e_a, d_a, f_a)) \), then the quality of leasing credit granting is “poor”.

Rule 2: If \( U((e_b, d_b, f_b)) = U((e_a, d_a, f_a)) \), then the quality of leasing credit granting is “fair”.

Rule 3: If \( U((e_b, d_b, f_b)) < U((e_a, d_a, f_a)) \), then the quality of leasing credit granting is “good”.

4. AN EXAMPLE

In this section, Jih Sun International Leasing and Finance Co., Ltd. of Taiwan is chosen as an example. The sample is one of the company’s clients selected by the leasing credit granting officer. By using the algorithm proposed in Section 3, we can determine the qualities of leasing credit granting.

Step 1: Use Eq.(2) to compute \( S_b \).

The volume of the client is “about 200 units”; sales is “about NT$50,000,000”; fixed cost is “about NT$5,000,000”; variable cost is “about NT$35,000,000”, which are provided by the leasing officer. The calculation unit of the breakeven interval and excess operating income is “NT$10,000” in the following computing.

Then, asking the leasing officer to approximate the fuzzy numbers of these variables, we obtain:

\( Q = \text{about 200} \approx (190, 200, 210) \),

\( S = \text{about 5000} = (4900, 5000, 5100) \) (unit:NT$10,000),

\( VC = \text{about 3500} = (3400, 3500, 3600) \) (unit:NT$10,000),

\( FC = \text{about 500} = (490, 500, 510) \) (unit:NT$10,000).

From the data, we obtain

\( V \equiv VC(\ast)Q = (16.19, 17.50, 18.95) \),

\( P \equiv S(\ast)Q = (23.33, 25.00, 26.84) \).

Then by using Eq.(2), we obtain

\( S_b \equiv FC(\ast)\{1(-)(\ast)\} \),

\( = (490, 500, 510)(\ast)[(16.19, 17.50, 18.95) \)(\ast)\}(23.33, 25.00, 26.84))\).

\( = (490, 500, 510)(\ast)[(0.60, 0.70, 0.81)]\)

\( = (490, 500, 510)(\ast)[(0.19, 0.30, 0.40)]\)

\( = (1225, 1666, 2688, 21)\).

Step 2: Calculate \( S_a \).

From the questionnaire, the leasing officer offers the expected values of the variables and transforms them into fuzzy numbers by using linguistic values as shown in Figure 1.
For example, with the linguistic variable “growth rate” which values are: \{very low, low, medium, high, very high\} as shown in Figure 1, the leasing officer predicts the next year “growth rate” of unit price, such as its value is “medium”, then the corresponding fuzzy number is \((0.25,0.5,0.75)\). Others can be done in the similar way. Thus, we can obtain the values of growth rate \(\Delta P, \Delta S, \Delta FC\), and \(\Delta V\) for \(P, S, FC\), and \(V\), respectively, from the questionnaire as follows.

\[
\begin{align*}
\Delta P &= \text{medium} = (0.25,0.5,0.75), \\
\Delta S &= \text{medium} = (0.25,0.5,0.75), \\
\Delta FC_1 &= \text{low} = (0.0,0.25,0.5), \\
\Delta FC_2 &= \text{low} = (0.0,0.25,0.5), \\
\Delta VC_1 &= \text{high} = (0.5,0.75,1), \\
\Delta VC_2 &= \text{medium} = (0.25,0.5,0.75), \\
\Delta VC_3 &= \text{high} = (0.5,0.75,1).
\end{align*}
\]

Here, \(S\): sales volume in units, \(P\): unit price, \(FC\): total fixed cost, \(V\): average variable cost. \(FC_1\): the wage, \(FC_2\): the fixed expenditure of manufacture, \(VC_1\): the material cost, \(VC_2\): the expenditure of variable manufacture, \(VC_3\): the direct labor cost.

Let \(\Delta A_1\) be the fuzzy number of incremental breakeven interval after leasing. By using Eq. (2), the incremental breakeven interval is

\[
\begin{align*}
\Delta A_1 &= \{(0.0,0.25,0.5)\} \times \{(0.10,0.24,0.66)\} \times \{(0.25,0.5,0.75)\} \\
&= \{(0.0,0.25,0.75)\} \times \{(0.13,0.48,2.64)\} \\
&= \{(0.0,0.25,0.75)\} \times \{(0.04,0.15,0.81)\} \\
&= \{(0.0,0.25,0.75)\} \times \{(0.19,0.85,0.96)\} \\
&= \{(0.0,0.29,2.63)\}.
\end{align*}
\]

The normalized value of \(\Delta A\)

\[
(0.0,0.10,0.90).
\]

According to Eq. (6), we obtain

\[
S_n \equiv S \times (1(\Delta A_1)) \\
= (1225.00, 1666.67, 2684.21) \times (1, 1.1, 1.9) \\
= (1225.00, 1833.34, 5100.00).
\]

**Step 3: Determine the \(R^1\) and \(R^2\).**

By the leasing officer’s prediction, we can obtain the multiple of net sales next year by comparing with breakeven point of this year. To obtain the values of \(r_{11}, r_{12},\) and \(r_{13}\), for instance, we ask: *what is the multiple of expected net sales next year in "good/fair/poor" state, respectively, if current year the client's breakeven point is in "good" state.*

Here, “good/fair/poor” state means that (1) the excess operating income of client is “greater/equal/smaller” than zero, and (2) the sales growth rate of client is “greater/equal/smaller” than average sales growth rate of the industry, respectively.

Others can be done in the similar way. Then from the questionnaire, the reward matrix is

\[
\frac{1}{16} \begin{bmatrix}
1655.34 & 2166.67 & 5526.90 \\
1.35 & 1.33 & 2.38 \\
1.38 & 1.33 & 2.59 \\
1.41 & 2.00 & 3.25 \\
1.38 & 1.33 & 2.59 \\
997 & 1000 & 1627.66
\end{bmatrix}
\]

Suppose that \(r_{ij} = r_{ij}^2 \quad (\forall i,j = 1,2,3)\).
According to Eqs.(7) and (8), the excess operating income matrices before leasing $R^1$ and after leasing $R^2$ are:

$$R^1 = \begin{bmatrix}
    r_{11} (-S_b) & r_{12} (-S_b) & r_{13} (-S_b) \\
    r_{21} (-S_b) & r_{22} (-S_b) & r_{23} (-S_b) \\
    r_{31} (-S_b) & r_{32} (-S_b) & r_{33} (-S_b) \\
\end{bmatrix}$$

$$= \begin{bmatrix}
    -1078.87,500.00,1325.84 \quad -1325.84,166.67,6759.04 \quad -1696.31,333.33,945.21 \\
    -1140.61,416.66,2165.96 \quad -1325.84,166.67,6759.04 \quad -1202.36,333.33,2030.32 \\
    -1202.36,333.33,2030.32 \quad -1325.84,166.67,6759.04 \quad -1943.28,666.67,402.66 \\
\end{bmatrix}$$

$$R^2 = \begin{bmatrix}
    r_{11} (-S_b) & r_{12} (-S_b) & r_{13} (-S_b) \\
    r_{21} (-S_b) & r_{22} (-S_b) & r_{23} (-S_b) \\
    r_{31} (-S_b) & r_{32} (-S_b) & r_{33} (-S_b) \\
\end{bmatrix}$$

$$= \begin{bmatrix}
    -1374.63,01795.04 \quad -4112.10, 0.00, 945.21 \\
    -3618.15,466.66,2030.32 \quad -3741.63,01759.04 \quad -4339.07,483.34,402.66 \\
\end{bmatrix}$$

where $S_b = (1225.00, 1666.67, 2684.21)$, $S_x = (1225.00, 1833.34, 5100.00)$.

**Step 4 : Determine the $P^1$ and $P^2$.**

By asking the leasing officer to predict the transition probability, we can obtain the $P^1$ data. To obtain the $p_{11}^1$, $p_{12}^1$, and $p_{13}^1$, for instance, we ask: what is the transition probability next year in "good/fair/poor" state, respectively, if current year the client's breakeven point is in "good" state. Others can be done in the similar way. Thus, the transition probabilities over a 1-year period from one state to another can be in terms of the following Markov chain:

$$P^1 = \begin{bmatrix}
    0.7 & 0.2 & 0.1 \\
    0.8 & 0.1 & 0.1 \\
    0.6 & 0.3 & 0.1 \\
\end{bmatrix}$$

We suppose that the transition probabilities before and after leasing are the same, i.e., assume that the business condition is unchanged in 1-year period, thus, the fuzzy transition probabilities matrices are

$$P^1 = P^2 = \begin{bmatrix}
    (0.7,0.7,0.7) & (0.2,0.2,0.2) & (0.1,0.1,0.1) \\
    (0.8,0.8,0.8) & (0.1,0.1,0.1) & (0.1,0.1,0.1) \\
    (0.6,0.6,0.6) & (0.3,0.3,0.3) & (0.1,0.1,0.1) \\
\end{bmatrix}$$

**Step 5: Calculate $u^k$**

According to Eq.(5), we can calculate the excess operating income $u^1$ before leasing.

If the state of breakeven is "good" before leasing, then the excess operating income of next year $u^1_1$ is:

$$u^1_1 = [0.7(-1078.87)+0.2(-1325.84)+0.1(-1696.31),
0.7(500)+0.2(166.67)+0.1(-333.33),
0.7(1325.84)+0.2(1759.04)+0.1(945.21)]
= (-1190.01, 350.00, 1734.42).$$

If the state of breakeven is fair before leasing, then the excess operating income next year $u^1_2$ is:

$$u^1_2 = [0.8(-1140.61)+0.1(-1202.36)+0.1(-1325.84),
0.8(416.66)+0.1(333.33)+0.1(166.67),
0.8(2165.96)+0.1(2030.32)+0.1(1759.04)]
= (-1165.31, 316.66, 2111.70).$$

If the state of breakeven is poor before leasing, then the excess operating income next year $u^1_3$ is:

$$u^1_3 = [0.6(-1202.36)+0.3(-1325.84)+0.1(-1943.28),
0.6(333.33)+0.3(166.67)+0.1(-666.67),
0.6(2030.32)+0.3(1759.04)+0.1(402.66)]
= (-1313.50, 183.33, -1969.50).$$

Similarly, according to Eq.(5), we can calculate the excess operating income $u^2$ after leasing.

If the state of breakeven is "good" after leasing, then the excess operating income of next year $u^2_1$ is:

$$u^2_1 = [0.7(-3494.66)+0.2(-3741.63)+0.1(-4112.10),
0.7(333.33)+0.2(0)+0.1(-500),
0.7(2301.6)+0.2(1759.04)+0.1(945.21)]
= (-1190.01, 13.33, 2057.45).$$
Fuzzy Set in Leasing Credit Granting

If the state of breakeven is "fair" after leasing, then the excess operating income of next year \( u^2 \) is:

\[
\begin{align*}
  u_2^1 &= [0.8(-3556.4)+0.1(-3618.15)+0.1(-3741.63), \\
         & \quad 0.8(249.99)+0.1(166.66)+0.1(0), \\
         & \quad 0.8(2165.96)+0.1(2033.32)+0.1(1759.04) \\
   &= (-351.10, 216.66, 2112.00).
\end{align*}
\]

If the state of breakeven is "poor" after leasing, then the excess operating income of next year \( u_3^2 \) is:

\[
\begin{align*}
  x_1 &= -1190.01, \text{ and} \\
  x_2 &= 2057.45, \text{ then}
\end{align*}
\]

\[
\begin{align*}
  u_3^2 &= [0.6(-3618.15)+0.3(-3741.63)+0.1(-4359.07), \\
         & \quad 0.6(166.66)+0.3(0)+0.1(-833.34), \\
         & \quad 0.6(2030.32)+0.3(1759.04)+0.1(402.66) \\
   &= (-3729.29, 16.66, 1786.17).
\end{align*}
\]

**Step 6: Determine the quality of leasing credit granting.**

By using Eq. (3) to rank fuzzy numbers, we can determine the quality of leasing credit granting of the client. From **Step 5**, we have:

\[
\begin{align*}
  u_i^1 &= (-1190.01, 350.00, 1374.42), \text{ and} \\
  u_i^2 &= (-1190.01, 183.33, 2057.45).
\end{align*}
\]

By Eq. (3), we have

\[
\begin{align*}
  U_T(u_i^1) &= \left\{ \left( 1374.42 + 1190.01 \right) / \\
  & \quad (2057.45 + 1190.01 - 350 + 1374.42) \\
  & \quad + \left[ 1 - \left( 2057.45 + 1190.01 \right) / \\
  & \quad (2057.45 + 1190.01 + 350 + 1190.01) \right] \right\} / 2 \\
  &= 0.46
\end{align*}
\]

\[
\begin{align*}
  U_T(u_i^2) &= 0.47.
\end{align*}
\]

The client's operating excess income and ranking values before and after leasing are shown in Table 1 and Table 2.

**Table 1. The fuzzy numbers of excess operating income of the client before and after leasing**

<table>
<thead>
<tr>
<th>State of breakeven</th>
<th>Before leasing</th>
<th>After leasing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good</td>
<td>(-1190.01, 350.00, 1374.42)</td>
<td>(-1190.01, 183.33, 2057.45)</td>
</tr>
<tr>
<td>Fair</td>
<td>(-1165.31, 316.66, 2111.70)</td>
<td>(-3581.10, 216.66, 2112.00)</td>
</tr>
<tr>
<td>Poor</td>
<td>(-1313.50, 183.33, 1909.50)</td>
<td>(-3729.29, 16.66, 1786.17)</td>
</tr>
</tbody>
</table>

**Table 2. The ranking values of fuzzy numbers of the client’s excess operating income before and after leasing**

<table>
<thead>
<tr>
<th>State of breakeven</th>
<th>Before leasing</th>
<th>After leasing</th>
<th>The quality of leasing credit granting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good</td>
<td>0.46</td>
<td>0.47</td>
<td>Good</td>
</tr>
<tr>
<td>Fair</td>
<td>0.65</td>
<td>0.58</td>
<td>Poor</td>
</tr>
<tr>
<td>Poor</td>
<td>0.65</td>
<td>0.56</td>
<td>Poor</td>
</tr>
</tbody>
</table>

98/PanPacific Management Review, February 2001
According to the Table 2, Jin Sun Co. should accept the client and lease to him. Only in the “good” state of breakeven before leasing, because its operating performance is improved after leasing, i.e., the quality of leasing credit granting is “good”. While those of “fair” and “poor” states aren’t improved after leasing, their qualities of leasing credit granting are “poor”, so Jin Sun Co. should refuse their leasing contracts except “good” state of breakeven.

5. CONCLUSIONS

This study offers a leasing credit granting model that adopts the fuzzy theory to deal with the uncertainty of variables in cost-volume-profit analysis to calculate the fuzzy number of breakeven interval of every single client to secure leasing credit. From Step1-2 in Section 4 and by Eq.(3), we find that $U_T(S_a) > U_T(S_b)$, i.e., the ranking value of breakeven interval after leasing is greater than that of breakeven interval before leasing. It also means that the credit evaluation is fulfilled "safety-first oriented rule"[15] to secure the leasing credit granting decision-making. Then the author attempts to combine fuzzy breakeven analysis with the transition matrix in the Markov chain to compute the operating performance before and after leasing for the client to monitor the quality of credit granting.

An algorithm for leasing credit granting quality is proposed in Section 3. An real example is discussed in Section 4 to show the practical application of the algorithm. By using Kim and Park’s ranking method, the ranking values of fuzzy numbers of client’s excess operating incomes can see if the client’s operating performance is improved or not. The leasing corporate can also distinguish the “good” credit from “bad” credit based on our proposed method. According to Section 4, we also find that the quality of leasing credit is "improved" only in the client’s "good" state of breakeven before leasing in the case of Jin Sun Co., while those of "fair" and "poor" state aren’t improved after leasing. An important aspect of the example shown in Section 4 is that it can be considered as the starting point for applications of fuzzy breakeven analysis in leasing management. Two points of future work are (1) to collect comprehensive sample from different industries to see the leasing policy of leasing companies, and (2) to consider many stages (years) to monitor and secure the quality of leasing credit granting decision-making.

ACKNOWLEDGEMENTS

The author gratefully acknowledges the careful reading and constructive suggestions of the referees which led to the improvement of the paper. The author also thanks Miss I. L. Chiang for her help in the preparation of this paper. Finally, the author wishes to thank Prof. T. H. Hsu for his continuous supporting me to complete the needed manuscript.

99/PanPacific Management Review, February 2001
REFERENCES


