Optimum Demodulation Codes for BPSK/QPSK

DS-SS Systems under Multiple Narrowband

Interference

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ABSTRACT

Interference suppression is the inherent capability of a direct-sequence spread spectrum (DS-SS) communication system. Due to bandwidth restriction or strong interference, however, the processing gain of original PN code may fail to reject the interference effectively. Instead of the original PN code, both optimum demodulation codes for the BPSK/DS-SS and QPSK/DS-SS systems are established in the presence of multiple narrowband interference. Moreover, the analytical results of optimum demodulation codes for both systems are derived. In addition, FIR transversal filters based on the LMS algorithm are constructed to search both the optimum demodulation codes. Simulation results show that the optimum demodulation codes significantly outperform the original PN code demodulation.

(Key words: Direct sequence spread spectrum, BPSK, QPSK, Multiple interference)
1. INTRODUCTION

Spread spectrum techniques have been early applied in military communications to enhance communication performance and to prevent enemy from interception due to the ability of anti-jamming [1]. Moreover, the inherent processing gain of a spread spectrum provides the system with an interference rejection capability [2], [3]. However, the processing gain does not provide sufficient improvement due to the bandwidth restriction or stronger narrowband interference. To overcome this problem, the past researches of [4]-[7] used extra signal processing techniques before despreading at the receiver. Previous work we mentioned above follows primarily the same rule that how BPSK direct sequence spread spectrum (BPSK/DS-SS) system performance can be improved by using extra signal processing techniques under single-tone interference. In [8], a transversal filter in BPSK/DS-SS system and a complex transversal filter in QPSK/DS-SS system under multiple narrowband interference were investigated. In the recent paper [9], it offered a different method from a traditional spread spectrum method that used the same PN sequence at the transmitter and the receiver. It found the optimum demodulation code, instead of the original PN sequence, can despread the received signal and reject interference simultaneously in a BPSK/DS-SS system. Thus, the output signal to interference plus noise ratio (SINR) of this method is larger than that of using the original PN sequence.

The interference is just a single-tone signal in [9]. However, smart jamming are probably multiple-tone. In this paper, an optimum demodulation code for a BPSK/DS-SS system and a complex optimum demodulation code for QPSK/DS-SS system are established. In addition, based on the LMS algorithm and the complex LMS algorithm [10], we also construct a FIR transversal filter to search optimum demodulation codes for the two systems.

In Section II, we derive the analytical results of optimum demodulation code for BPSK/DS-SS receiver. In Section III, the closed-form expressions of complex optimum demodulation code for QPSK/DS-SS receiver are established. Section IV employs the FIR transversal filters based on the LMS algorithm to recursively calculate the optimum demodulation code vectors. Numerical results are simulated in Section V and a brief conclusion is presented in Section VI.
II. OPTIMUM DEMODULATION CODE FOR
BPSK/DS-SS RECEIVER

A. BPSK/DS-SS SYSTEM MODEL

As shown in Fig. 1(a), the transmitted signal of a BPSK/DS-SS system can be expressed as

\[ s(t) = Ab(t)c(t) \cos \omega_c t \]  \hspace{1cm} (1)

where \( A \) is carrier amplitude, the data bit signal \( b(t) \in \{-1,1\} \) is spread by a PN sequence \( c(t) \). Then, the received signal at the receiver, as shown in Fig. 1(b), is given by

\[ r(t) = s(t) + J(t) + n_w(t) \]  \hspace{1cm} (2)

where the \( M \) multiple narrowband interference is

\[ J(t) = \sum_{k=1}^{M} \alpha_k \cos[(\omega_c + \Omega_k) t + \theta_k] \]. \hspace{1cm} (3)

\( \alpha_k \) is the amplitude of the \( k \)th interference. \( \Omega_k \) is the frequency offset between the interference and the carrier. \( \theta_k \) is the random phase uniformly distribute in \([0,2\pi]\). The additive white gaussian noise (AWGN) \( n_w(t) \) is

\[ n_w(t) = n_r(t) \cos \omega_c t + n_q(t) \sin \omega_c t \]  \hspace{1cm} (4)

Through the carrier match filter, as shown in Fig. 1(b), the baseband received signal at the \( n \)th chip of the \( m \)th bit is given by

\[ x_{m,n} = d_{m,n} + J_{m,n} + n_{m,n} \] \hspace{1cm} (5)

where

\[ d_{m,n} = AT_c b_m c_n \]

\[ J_{m,n} = \sqrt{J} \sum_{k=1}^{M} \cos[\Omega_k (mL + n)T_c + \phi_k] \]
\[ \phi_k = \theta_k - \frac{\Omega_k T_c}{2} \]
\[ n_{m,n} = n_1 (mL T_c + nT_c) \]

\( J \) is the average power of narrowband interference. \( L \) is the length of optimum demodulation code. First term in eqn. (5) is spread data signal. Second term is multiple narrowband interferers whose average powers are assumed equally. \( n_1 \) is the in-phase component of \( n_0(t) \) passed through the carrier match filter with zero mean and average power of \( \sigma_n^2 \).

**B. OPTIMUM DEMODULATION CODE FOR BPSK/DS-SS SYSTEM**

Define the received signal vector per bit \( x_m = [x_{m,L-1}, x_{m,L-2}, \ldots, x_{m,0}]^T \) and the demodulation code vector \( w = [w_{L-1}, w_{L-2}, \ldots, w_0]^T \), where the subscript \( T \) denotes transpose.

Furthermore, we define the error function \( e_m \) between the reference bit sequence \( r_m \in \{-1,1\} \) and the input signal at decision circuit as
\[ e_m = r_m - g^T x_m \]

where \( g = w / L \). Then, the mean-square error (MSE) is
\[ \varepsilon = E[|e_m|^2] = 1 - 2g^T p_m + g^T R_m g \]

where the cross-correlation vector \( p_m = E[r_m x_m] \), the auto-correlation matrix \( R_m = E[x_m x_m^T] \). For minimizing the MSE \( \varepsilon \), we let the first-order differential of eqn. (7) with respect to \( g \) equal to zero, that is,
\[ \frac{\partial \varepsilon}{\partial g} = -2p_m + 2R_m g = 0 \]
According to eqns. (6) and (8), we can obtain the Wiener-Hopf equation for optimum demodulation code of BPSK DS-SS system as

$$\mathbf{w}_{\text{optBPSK}} = L\mathbf{R}_m^{-1}\mathbf{p}_m$$

(9)

To simplify derivation, we define spread data signal vector $\mathbf{d}_m$, narrowband interference signal vector $\mathbf{J}_m$, AWGN vector $\mathbf{n}_m$ and PN sequence vector $\mathbf{c}$ as followings:

$$\mathbf{d}_m = [d_{m,L-1}, d_{m,L-2}, \ldots, d_{m,0}]^T$$

(10)

$$\mathbf{J}_m = [J_{m,L-1}, J_{m,L-2}, \ldots, J_{m,0}]^T$$

(11)

$$\mathbf{n}_m = [n_{m,L-1}, n_{m,L-2}, \ldots, n_{m,0}]^T$$

(12)

$$\mathbf{c} = [c_{L-1}, c_{L-2}, \ldots, c_0]^T$$

(13)

Then, the discrete baseband signal vector can be expressed as

$$\mathbf{x}_m = \mathbf{d}_m + \mathbf{J}_m + \mathbf{n}_m$$

(14)

Since the spread data signal and narrowband interference are mutually independent [11], the auto-correlation matrix is

$$\mathbf{R}_m = E[\mathbf{x}_m \mathbf{x}_m^T] = \mathbf{R}_d + \mathbf{R}_j + \mathbf{R}_n$$

(15)

where

$$\mathbf{R}_d = E[\mathbf{d}_m \mathbf{d}_m^T] = A^2 T_c^2 \mathbf{c}\mathbf{c}^T$$

(16)
\[
\mathbf{R}_n = E[\mathbf{n}_n \mathbf{n}_n^T] = \frac{\sigma_n^2}{2} \mathbf{I}_{L \times L}
\]

Similarly, we can get the closed-form expression of the cross-correlation vector \( \mathbf{p}_m \) as

\[
\mathbf{p}_m = E[r_m x_m] = \mathbf{A} \mathbf{T}_c \mathbf{c}
\]

Substituting eqns. (10)–(19) in eqn. (9), we can obtain the analytical results of the optimum demodulation code \( \mathbf{w}_{opt/\text{BPSK}} \).

III. COMPLEX OPTIMUM DEMODULATION CODE

FOR QPSK/DS-SS RECEIVER

A. QPSK/DS-SS SYSTEM MODEL

Complex optimum demodulation code is applied in QPSK/DS-SS system, as shown in Fig. 2(a). The data signal, which is spread at the transmitter, may be expressed as [11]

\[
s'(t) = A b_1(t) c_1(t) \cos \omega t - A b_2(t) c_2(t) \sin \omega t
\]

(20)

where \( c_1(t) \) and \( c_2(t) \) are uncorrelated PN sequence. In the presence of multiple narrowband interference, the received signal at the receiver, as shown Fig. 2(b), is given by

\[
r'(t) = s'(t) + J(t) + n_w(t)
\]

(21)

Through the carrier match filter, the complex representation of the baseband received
signal at the \( n \)th chip of the \( m \)th bit is given by

\[
x'_{m,n} = x_{1m,n} + jx_{2m,n} \quad n = 0,1,\ldots,L-1
\]

where

\[
x_{1m,n} = d_{1m}c_{1,n} + \sqrt{J} \sum_{k=1}^{M} \cos[\Omega_k (mL + n)T_c + \phi_k] + n_{1m,n}
\]

\[
x_{2m,n} = d_{2m}c_{2,n} + \sqrt{J} \sum_{k=1}^{M} \sin[\Omega_k (mL + n)T_c + \phi_k] + n_{2m,n}
\]

\[
d_{1m} = \pm AT_c = d_{2m},
\]

\[
n_{1m,n} = n_1(mLT_c + nT_c), \quad \text{and} \quad n_{2m,n} = n_2(mLT_c + nT_c)
\]

**B. COMPLEX OPTIMUM DEMODULATION CODE FOR QPSK/DS-SS RECEIVER**

Define the received signal vector per symbol

\[
x'_m = [x'_{m,L-1}, x'_{m,L-2}, \ldots, x'_{m,0}]^T
\]

and the complex demodulation code vector

\[
w' = [w'_{L-1}, w'_{L-2}, \ldots, w'_0]^T
\]

Similar to the derivation of eqn. (9), we can get the optimum complex optimum code as

\[
w_{opt,QPSK} = LR'_{m,-1}p'_m
\]

To simplify derivation, we define spread data signal vector \( d'_m \), narrowband interference signal vector \( J'_m \), AWGN vector \( n'_m \), PN sequence vectors \( c_1 \) and \( c_2 \) as followings:

\[
d'_m = [d'_{m,L-1}, d'_{m,L-2}, \ldots, d'_{m,0}]^T = d_{1m}c_1 + jd_{2m}c_2
\]

\[
c_1 = [c_{1,L-1}, c_{1,L-2}, \cdots, c_{1,0}]^T, \quad c_2 = [c_{2,L-1}, c_{2,L-2}, \cdots, c_{2,0}]^T
\]

\[
J'_m = [J'_{m,L-1}, J'_{m,L-2}, \cdots, J'_{m,0}]^T
\]

\[
n'_m = [n'_{m,L-1}, n'_{m,L-2}, \cdots, n'_{m,0}]^T
\]
where \( J'_{m,L-1} = \sqrt{J} \sum_{k=1}^{M} e^{j[(mL+L-1)\Omega_i T_c + \phi_i]} \) and \( n'_{m,L-1} = n'_{1m,L-1} + jn'_{2m,L-1} \). Then, the discrete baseband signal vector can be expressed as
\[
x'_m = d'_m + J'_m + n'_m
\]
Therefore, the auto-correlation matrix is
\[
R'_m = E[x'_m x'_m^H] = R'_d + R'_f + R'_n
\]  
(33)
where
\[
R'_d = E[d'_m d'_m^H] = A^2 T_c^2 [c_1 c_1^T + c_2 c_2^T]
\]  
(34)
\[
R'_f = E[J'_m J'_m^H]
\]
(35)
\[
R'_n = E[n'_m n'_m^T] = \sigma_n^2 I_{L \times L}
\]  
(36)
Similarly, we can simplify the cross-correlation vector \( p'_m \) as
\[
p'_m = E[r'_m^* x'_m] = A T_c [c_1 + c_2]
\]  
(37)
where the reference symbol \( r'_m = a_1 + ja_2 \) and \( a_1, a_2 \in \{1,-1\} \). Substituting eqns. (28)–(37) in eqn. (27), we can obtain the analytical results of the complex optimum demodulation code \( w_{\text{opt/QPSK}} \).
IV. RECURSIVE SEARCH FOR OPTIMUM DEMODULATION CODE BASED ON LMS ALGORITHM

We must know the auto-correlation $R_m$ and cross-correlation $P_m$ in advance to calculate the optimum demodulation code based on Wiener-Hopf equation. In addition, we have to calculate the inverse matrix of auto-correlation matrix. Therefore, we design a FIR transversal filter, as shown in Fig. 3, based on LMS algorithm [10] to calculate the estimate optimum demodulation code vector for the BPSK/DS-SS receiver, that is,

$$e_m = r_m - g_m^T x_m$$

(38)

$$g_{m+1} = g_m + \mu e_m x_m$$

(39)

where $e_m$ is an error function between the $m$th reference training bit $r_m$ and the input signal of decision circuit, $g_m$ is the estimate weight vector at $m$th bit, $\mu$ is a step-size parameter. If and only if the step-size parameter $\mu$ satisfies the condition $0 < \mu < \frac{2}{\lambda_{\text{max}}}$, then the LMS algorithm is convergent, where $\lambda_{\text{max}}$ is the largest eigenvalue of the auto-correlation matrix $R_m$ [10]. Here, the value of $\mu$ is related to convergent rate. As $m \to \infty$, LMS algorithm is convergent. Hence, $g_m$ will be convergent to optimum solution $w_{\text{opt/BPSK}} / L$.

Similarly, based on the complex LMS algorithm, we can recursively calculate the estimate weight vector $g'_m$ which is convergent to optimum solution $w_{\text{opt/QPSK}} / L$ for the QPSK/DS-SS receiver, that is,

$$e'_m = r'_m - g'_m^T x'_m$$

(40)

$$g'_{m+1} = g'_m + \mu e'_m x'_m$$

(41)
V. NUMERICAL RESULTS

Let the normalized frequency offset $\Omega_N = \Omega / 2\pi$. To explore the frequency response under two-tone interferers, we assume that average power of each narrowband interference to the reference signal ratio JSR=20 dB, the desired signal energy per bit to the noise ratio SNR=10 dB, the PN code length $L=31$, $\Omega_{N,1} = 0.025$, $\Omega_{N,2} = 0.125$. Fig. 4 shows the frequency response of the two receivers. Like a notch filter, both the optimum demodulation codes can accurately reject the two-tone interferers. Apparently, BPSK/DS-SS receiver outperforms QPSK/DS-SS receiver in interference suppression. Moreover, the frequency response of BPSK/DS-SS receiver near the high frequency offset is better than that of QPSK/DS-SS receiver. Therefore, based on the optimum demodulation code, we can expect BPSK/DS-SS receiver performs better than QPSK/DS-SS receiver.

For performance comparison, we define the output MSE$_{\text{BPSK}}$, MSE$_{\text{QPSK}}$ and MSE$_{\text{PN}}$ as

$$
\text{MSE}_{\text{BPSK}} = E[|r_m - \frac{1}{L} \sum_{n=0}^{L-1} w_{\text{opt,BPSK}} x_{m,n}|^2]
$$

$$
\text{MSE}_{\text{QPSK}} = E[|r'_m - \frac{1}{L} \sum_{n=0}^{L-1} w_{\text{opt,QPSK}} x'_{m,n}|^2]
$$

$$
\text{MSE}_{\text{PN}} = E[(a_1 - \frac{1}{L} \sum_{n=0}^{L-1} x_{1m,n} c_{1,n})^2 + (a_2 - \frac{1}{L} \sum_{n=0}^{L-1} x_{2m,n} c_{2,n})^2]
$$

Fig. 5 shows the MSE under two-tone interference for $\Omega_{N,1} = 0.025$ and $\Omega_{N,2}$ varies between 0--0.5. Clearly, the original PN code demodulation breaks down. Moreover, MSE$_{\text{BPSK}}$ is less than MSE$_{\text{QPSK}}$ as we expect. Fig. 6 plots the BER performance comparison in the same scenario of Fig. 5. BPSK/DS-SS receiver still performs best.

Fig. 7 shows the average 500 times MSE$_{\text{QPSK}}$ based on complex LMS algorithm with 2000 iterations for JSR=20 dB, SNR=10 dB, $\Omega_N = 0.025$, $\mu = 0.003$. It is shown that when MSE is convergent to the optimum value as $m$ approaches about 1500. However, the convergence value is related to the value of $\mu$. The smaller $\mu$ induces less misadjustment, but the convergence rate is slower [10]. Accordingly, without the statistical estimation of autocorrelation matrix and cross-correlation vector, the complex optimum demodulation code
can be calculated recursively based on the complex LMS algorithm.

VI. CONCLUSION

To simultaneously suppress multiple narrowband interference and despread received signal, we develop optimum demodulation codes for the BPSK/DS-SS and QPSK/DS-SS receivers. Closed-form expressions for the optimum demodulation codes are derived. The advantages of the optimum demodulation codes based on Wiener-Hopf equation are the following: 1) Extra signal processing techniques are not required to reject multiple narrowband interference. 2) Original PN code is not needed to despread the received signal. As the PN code demodulation breaks down in presence of multiple narrowband interference, the optimum demodulation codes still work properly. Moreover, we construct FIR transversal filters based on LMS algorithm to search the optimum demodulation code vector. The advantage of the recursive algorithm is that the statistical estimation in Wiener-Hopf equation is not required. However, the method requires reference training bits.

REFERENCES


Fig. 1(a). BPSK/DS-SS transmitter system

Fig. 1(b). BPSK/DS-SS receiver system
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Fig. 2(a). QPSK/DS-SS transmitter system

Fig. 2(b). QPSK/DS-SS receiver system
Fig. 3. FIR transversal filter based on LMS algorithm
Optimum Demodulation Codes for BPSK/QPSK
DS-SS Systems under Multiple Narrowband Interference

Fig. 4. The frequency response under tow-tone jammers, $\Omega_{N,1} = 0.025$ and $\Omega_{N,2} = 0.125$

Fig. 5. MSE performance comparison under tow-tone jammers, JSR=20dB, SNR=10 dB
Fig. 6. BER performance comparison under tow-tone jammers, JSR=20dB, SNR=10 dB

Fig. 7. The learning curve of mean-square error based on complex LMS algorithm