Initial Capacity Decision in an Uncertain Product Market

RUEY-JI GUO
Department of Accounting, Soochow University

ABSTRACT

The uncertainty of demand for products has been a prominent issue in numerous management decisions, especially in the initial stage of providing a new product. Whether providing merchandise or a service, capacity decisions face large variances of demand. This paper addresses the impact of demand uncertainty on initial capacity decisions in the early period of providing a new product. Using a two-stage decision model, this paper finds that the optimal initial capacity level in stage one would ascend as demand variance increases if the cost of adding the capacity in stage two is relatively insignificant; otherwise, it would descend as demand uncertainty enlarges.

Keywords: capacity planning, cost management, demand uncertainty

INTRODUCTION

Demand uncertainty is prevalent in most management decisions and has a significant impact on the consequences of those decisions as shown in the literature. Atamtürk and Zhang (2007) describe a two-stage robust optimization approach for solving network flow and design problems with uncertain demand. Hood et al. (2003) present a method considering uncertainty, using stochastic integer programming to find a tool set that is robust to changes in demand. Law and Au (1999) make use of a supervised feed-forward neutral network model to forecast Japanese tourist arrivals in Hong Kong. Carey (1998) addresses the issue of hospital bed capacity by considering the stochastic demand for United States hospitals. de Castro et al. (1997) develop a production planning model with stochastic demand, a constant defective rate and capacity constraints to analyze the production problem of a chocolate milk manufacturer. Subrahmanyan and Shoemaker (1996) create a model for optimal pricing and inventory
policies for retailers facing uncertain demand. Kasilingam (1995) uses a non-linear programming model to deal with product mix problems in the presence of alternate process plans under uncertain demand. Raman and Chatterjee (1995) examine pricing policies for a monopolist facing uncertain demand in a market characterized by dynamics on the demand side and/or on the cost side. Explicitly, all of these models incorporate the impact of demand uncertainty, and provide an insight into the implications of uncertainty on the variety of management decisions.

Bok et al. (1998) indicate that, while the rapid development of computer technologies enables companies to increase the amount of knowledge about their customers, the parameters characterizing demand remain hard to estimate. Geng and Jiang (2009) also pinpoint that strategic capacity planning plays an important role in improving business performance due to a number of reasons, including high capital investment costs, complex fabrication processes, rapid changes in technology and products, long lead times and the high cost of capacity increments, and highly uncertain demand. Hence, initial capacity planning becomes a complicated decision that requires the decision maker to prudently incorporate various factors in the capacity decision.

Banker and Hughes (1994) argue that only normal cost enters into pricing rules established at the time initial capacities are set, and the production manager only requires the expected demand from marketing to make initial capacity decision, along with knowledge of the distribution of random demand shocks and costs, as well as production parameters. In capacity decision, Banker and Hughes (1994) consider not only the non-recoverable cost of adding capacity before demand becomes known, but also the expected penalty-adjusted cost of doing so after demand becomes known. Nevertheless, their study does not allow possible adjustments of normal capacity when demand becomes more certain or stable. Since capacity adjustments and other related costs can have a considerable influence on capacity planning, this paper examines how demand variance affects the initial capacity decision, and provides a number of policy implications.

This paper uses two capacity decision models to analyze the related issue. One is for one-stage (single stage) capacity decisions; the other is for two-stage capacity decisions. As
with Banker and Hughes (1994), it is assumed that it is optimal for the decision maker to meet demand (without lost sales), and capacity could be added as necessary to meet unexpected demand when it becomes known, albeit at a cost penalty. Essentially, in one-stage capacity decisions, the decision maker only needs to make the marginal cost of adding initial capacity equal to the expected penalty-adjusted cost of exceeding the initial (normal) capacity. However, in two-stage capacity decisions, the decision maker needs to further think about the possible capacity adjustment costs at the beginning of stage two.

The main purpose of model two is to address the issue of initial (normal) capacity decisions in stage one while facing possible adjustments of normal capacity after demand becomes stable or certain. Hence, for simplicity, it is assumed the demand in stage two is equal to the actual (realized) demand in stage one, and it is reasonable to adjust normal capacity level to match the actual demand level in stage one. While such a setting cannot capture the real situation, it highlights how the decision-makers need to consider the change from an uncertain demand to a stable demand. Furthermore, although capacity decisions can depend on pricing decisions for a determination of expected demand, pricing decisions will become straightforward after optimal capacity has been determined. In other words, after obtaining the optimal capacity function conditional on expected demand (or product price), the decision maker can easily further determine an optimal product price to maximize total profit.\(^1\) To avoid unnecessary complications and focus on addressing the impact of demand variance on initial capacity decision, this paper does not address pricing issues. In next section, this paper characterizes two models used in this study. The related analyses and results are presented in Section 3. Finally, the conclusion is in Section 4.

THE MODEL

In this paper, it is assumed that there is a risk-neutral business planning to provide a new product through its branches or chain stores, facing an initial capacity decision related to the number of branches or chain stores. There are two models to be used for analyzing the initial capacity decision. The first is designed to deal with one-stage capacity decisions, and the

\(^1\) Refer to Banker and Hughes (1994) for the related discussion.
second is created to address two-stage capacity decisions. In model one, what the firm faces is simply an uncertain demand stage. All operating activities would be completed in that stage, and the demand for new product is assumed to follow a uniform distribution, i.e., \( \bar{Q} \sim UNI(Q, \overline{Q}) \), where \( \bar{Q} \) is the random demand for the new product, and \( Q \) (\( \bar{Q} \)) denotes the lower (upper) bound of demand. In the latter analysis, it is further assumed that \( Q_e \) is the expected demand for the new product, and \( \varepsilon = \overline{Q} - Q_e = Q_e - \underline{Q} \). Also, it is assumed that the initial normal capacity (\( Q_e \)) could be set up only at the beginning of the stage, and any demand over the normal capacity is worthwhile to satisfy, but needs much more flexible resource costs per unit (\( \theta \cdot v \)) than that under normal capacity (\( v \)), i.e., \( \theta \cdot v > v \) where \( \theta (>1) \) is penalty factor and \( v \) is the unit variable cost under normal capacity.\(^2\) In addition, \( Q_0 \) denotes the basic capacity, a minimum capacity level the firm has to set up if it decides to provide the new product. In that case, the firm needs to decide a capacity multiplier (\( k \), e.g. the number of branches or chain stores, and let \( Q_e = kQ_0 \) where \( k \geq 1 \). Meanwhile, this paper defines the capacity related costs (\( F \)) as \( F_0 + a(k-1)Q_0 \), where \( F_0 \) is the fixed costs related to basic capacity (\( Q_0 \)) and \( a \) is the cost coefficient of setting additional capacity above the basic one. Thus, the total costs for providing a new product (\( TC \)) become \( F_0 + a(k-1)Q_0 + vQ \) if \( Q \leq Q_e \), but become \( F_0 + a(k-1)Q_0 + vQ_e + \theta(Q - Q_e) \) if \( Q > Q_e \), where \( Q \) is the actual (realized) demand of new product.

In model two, the possible capacity adjustment would be taken into account and allowed to happen at the beginning of stage two. It is assumed the firm would first face an uncertain demand stage (stage one), and then a stable demand stage (stage two). As in model one, in the uncertain demand stage, demand is assumed to follow a certain uniform distribution; i.e., \( \bar{Q}_1 \sim UNI(Q, \overline{Q}) \), where \( \bar{Q}_1 \) is the random demand in stage one, and \( Q (=Q_e - \varepsilon) \) as well as \( \overline{Q} (=Q_e + \varepsilon) \) denote the lower and upper bounds of demand for the new product, respectively. However, the demand in stage two, \( Q_2 \), is assumed to be certain and equal to the actual

\(^2\) The increased cost due to the actual demand over than the maximum capacity is called the “penalty cost” by Banker and Hughes (1994).
demand in stage one, $Q_1$, for simplicity. Therefore, if the initial normal capacity set up at the beginning of stage one (i.e., $Q_{c1}$) differs from the actual demand in stage one (or the expected demand in stage two), it would be necessary to adjust the normal capacity at the beginning of stage two. Meanwhile, it is assumed that any demand over the normal capacity is worthwhile to satisfy, but needed much more flexible resource costs per unit than that under normal capacity, i.e., $\theta \cdot v > v$ where $\theta (>1)$ is the penalty factor, and $v$ is the unit variable cost under normal capacity. Thus, at the beginning of stage one, the decision maker would have to take the possible costs of normal capacity adjustments in stage two into account, and set up an optimal initial (normal) capacity level, $Q_{c1}^*$. Then, in stage two, the stable demand stage, the normal capacity level $Q_{c2}$ could be adjusted from $Q_{c1}^*$ to the actual demand level in stage one, $Q_1$.

Intuitively, if the basic (minimum) capacity, $Q_0$, exceeds the lower demand bound, $Q$, it would become much more possible for the decision maker to give up producing the product due to a higher capacity cost. Hence, for simplicity, this paper considers only the case of $Q_0 < Q < Q$ in a two-stage model. Also, it is assumed $Q_{c1} = k_1 Q_0$ and $Q_{c2} = Q_1 = k_2 Q_0$, where $k_1$ and $k_2$ are the coefficients of the capacity level for stage one and stage two, respectively. Since $Q_0$ is the basic capacity level, both $k_1$ and $k_2$ should not be less than one, and $k_2 \neq k_1$ if $Q_1 \neq Q_{c1}$. In model two, $a$ is still the cost coefficient of setting additional capacity above the basic capacity, but $m$ and $d$ are used to denote the cost coefficients of maintaining and reducing capacity, respectively. Provided the fixed cost (or committed resource cost) in stage one ($F_1$) is $F_0 + a(k_1-1)Q_0$, the fixed cost in stage two ($F_2$) will be $mk_1 Q_0 + a(k_2-k_1)Q_0$ if $k_2 \geq k_1$, but will be $mk_2 Q_0 + d(k_1-k_2)Q_0$ if $k_2 < k_1$. The first term in $F_2$ is the capacity maintaining cost (for continuing to use the capacity set up in stage one), and the second term is the capacity adjusting cost (for adding additional capacity or dropping redundant capacity in stage two). In model two, it is assumed it is certain and equal to the demand in stage one.

3 While the demand in stage two could still be uncertain but have less variance, without losing analytical insight, it is assumed it is certain and equal to the demand in stage one.

4 As mentioned in the latter, if $d < m < a < (\theta - 1)v$, it is worthwhile for the firm to adjust up (or down) normal capacity at the beginning of stage two. Please refer to note 6.

5 Kaplan and Atkinson (1998) referred to the variable costs as “flexible resource costs,” and the fixed costs as “committed costs.”
stage two).

Obviously, if the related capacity is set up at the second stage, there will be only capacity setting costs incurred in that period, and there are no capacity maintaining costs. However, if the related capacity is set up in the first stage, there will be either capacity maintaining costs or capacity reducing costs incurred in the second period, depending on if the related capacity will be kept or dropped. Moreover, it is assumed that $a > m$.

To assure the capacity adjustment in stage two is economically desirable, it is assumed that $d < m < a < (\theta - 1)v$. Otherwise, it would be unnecessary to deal with the related capacity decision. In model two, a discount factor $(\alpha)$ is used, which is equivalent to $1/(1+\text{discount rate})$, to treat the related costs occurring in stage two.

**THE ANALYSIS**

This paper first demonstrates the results derived from model one, and then those derived from model two. As mentioned in the preceding section, in one-stage capacity decisions, the total costs for providing a new product ($TC$) are $F_0 + a(k-1)Q_0 + vQ$ if $Q \leq Q_c$, but become $F_0 + a(k-1)Q_0 + vQ_c + \theta(Q - Q_c)$ if $Q > Q_c$. Hence, the basic tension underlying one-stage capacity decisions is between the non-recoverable cost of increasing capacity before demand becomes known, and the expected penalty cost of doing so after demand becomes known. In the following propositions 1 to 3, the optimal initial (normal) capacity levels ($Q^*_c$) under three different assumptions regarding basic capacity ($Q_0$) are presented.

**Proposition 1:**

Under $a < (\theta - 1)v$, if $\bar{Q} \sim \text{UNI}(\underline{Q}, \bar{Q})$ and $Q_0 < \underline{Q} < \bar{Q}$, then the optimal initial (normal) capacity level $Q^*_c$ will be $\bar{Q} - 2a(\theta - 1)v$ where $v = (\bar{Q} - \underline{Q})/2$. Meanwhile, $\underline{Q} < Q^*_c < \bar{Q}$.

[Proof] See appendix A.

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6 Among others, $a < (\theta - 1)v$ is used to make it economically desirable for the decision maker to adjust the normal capacity up at the beginning of stage two under $Q_1 > Q_{c1}$, and $d < m$ is used to make it economically desirable for the decision maker to adjust the normal capacity down at the beginning of stage two under $Q_1 < Q_{c1}$. 
When the basic capacity \((Q_0)\) is less than the lower bound of possible demand for new product \((\bar{Q})\), using the result of proposition 1, it is optimal for the firm to set up an initial capacity level of \(\bar{Q} - 2a/(\theta - 1)\nu\), with the optimal capacity between minimum demand and maximum demand. In addition, the optimal capacity obviously depends on the expected demand, the cost coefficient of setting additional capacity, the normal unit variable cost, the penalty factor, and the possible demand range.

Proposition 2:

Under \(a < (\theta - 1)\nu\), if \(\tilde{Q} \sim \text{UNI}(\underline{Q}, \overline{Q})\) and \(\underline{Q} < Q_0 < \overline{Q}\), then

1. \(Q^*_c = \bar{Q} - 2a/(\theta - 1)\nu \geq Q_0\) if \(a \leq (\bar{Q} - Q_0)(\theta - 1)/2\nu\), but
2. \(Q^*_c = Q_0\) if \(a > (\bar{Q} - Q_0)(\theta - 1)/2\nu\).

Meanwhile, \(\underline{Q} < Q_0 \leq Q^*_c < \overline{Q}\).

[Proof] See appendix B.

When the basic capacity \((Q_0)\) is larger than minimum demand \((\underline{Q})\) but less than maximum demand \((\overline{Q})\), according to proposition 2, the optimal capacity level can be nothing but the basic capacity \((Q_0)\) if the cost coefficient of setting additional capacity above the basic capacity is relatively higher. Nevertheless, the optimal capacity level remains \(\overline{Q} - 2a/(\theta - 1)\nu\) if the cost coefficient \(a\) is relatively smaller.

Furthermore, as shown in proposition 3, when the basic capacity \((Q_0)\) is over the upper bound of demand \((\overline{Q})\), the basic capacity \((Q_0)\) will be the optimal capacity level as long as the cost coefficient, \(a\), is less than the increased (penalty) unit variable cost, \((\theta - 1)\nu\).

Proposition 3:

Under \(a < (\theta - 1)\nu\), if \(\tilde{Q} \sim \text{UNI}(\underline{Q}, \overline{Q})\) and \(\underline{Q} < \overline{Q} \leq Q_0\), then \(Q^*_c = Q_0\).

[Proof] See appendix C.

Since \(Q_0 < \underline{Q} < \overline{Q}\) is the case most possible in practice, this paper is interested in how demand uncertainty influences the optimal capacity level, i.e., \(\overline{Q} - 2a/(\theta - 1)\nu\). In corollary 1,
half a demand range \( \epsilon \) is employed as a proxy of demand uncertainty and its impact on optimal capacity is analyzed.

Corollary 1:

Under \( a < (\theta - 1)\nu \), if \( \bar{Q} \sim \text{UNI}(Q_e - \epsilon, Q_e + \epsilon) \) and \( Q_0 < \underline{Q} < \bar{Q} \) where \( \epsilon = (\bar{Q} - Q) / 2 \), then the optimal initial (normal) capacity level \( Q^*_c \) will ascend as \( \epsilon \) increases if \( a < (\theta - 1)\nu / 2 \), but will descend as \( \epsilon \) increases if \( a \geq (\theta - 1)\nu / 2 \).

[Proof] See appendix D.

As shown in corollary 1, the enlargement of demand uncertainty (\( \epsilon \)) will lead to an increase in optimal capacity (\( Q^*_c \)) if the condition of \( a < (\theta - 1)\nu / 2 \) is satisfied; otherwise, it will result in a decrease in optimal capacity. In other words, ceteris paribus, if the cost coefficient of setting additional capacity above the basic capacity is less than half the increased unit variable cost as actual demand over normal capacity, it is optimal for the firm to adjust its normal capacity up as demand uncertainty ascends.

However, the results from model one are subject to some constraints. Among others, model one does not allow for possible adjustments in normal capacity after a period of time. Therefore, this paper creates another model (model two) to deal with the issue of possible capacity adjustments. Specifically, the second model considers a possible capacity adjustment in stage two when demand becomes stable and certain. Based on the settings and assumptions regarding model two, mentioned in section 2, the related results are derived as shown in the following proposition and corollaries. To simplify the results, they are all derived from the assumption of a lower basic capacity, i.e., \( Q_0 < \underline{Q} < \bar{Q} \).

Proposition 4:

Under \( d < m < a < (\theta - 1)\nu \), if \( \bar{Q}_1 \sim \text{UNI}(\underline{Q}, \bar{Q}) \) and \( Q_2 = Q_1 \), then the optimal initial capacity level in stage one \( Q^*_c \) will be \( \bar{Q} - 2\epsilon(a + ad) / [(\theta - 1)\nu + a(a - m + d)] \) where \( \epsilon = (\bar{Q} - Q) / 2 \).

Furthermore, \( \underline{Q} < Q^*_c < \bar{Q} \).
Under the assumption of \( d < m < a < (\theta - 1)v \), proposition 4 shows the optimal initial (normal) capacity level in model two will depend on not only those factors mentioned in proposition 1 but also other factors such as the discount factor \((\alpha)\), the cost coefficient of maintaining capacity \((m)\), and the cost coefficient of reducing capacity \((d)\). Since there remains demand uncertainty in stage one, this paper is still interested in how demand variance influences the optimal initial capacity. In Corollary 2, it is shown that the impact of demand uncertainty on optimal capacity is dependent on the relationship between the cost coefficient of setting additional capacity above the basic capacity and other variables, including the discount factor \((\alpha)\), the cost coefficient of maintaining capacity \((m)\), and the cost coefficient of reducing capacity \((d)\). That is, the enlargement of demand uncertainty \((\varepsilon)\) will lead to an increase in optimal capacity \(Q^*_1\) if the condition of \( a < ((\theta - 1)v - \alpha(m + d))/(2 - \alpha) \) is satisfied; otherwise, it will result in a decrease in optimal capacity.

**Corollary 2:**

Under \( d < m < a < (\theta - 1)v \), if \( Q_1 \sim UNI(Q_e - \varepsilon, Q_e + \varepsilon) \) and \( Q_2 = Q_1 \), then the optimal initial capacity level in stage one \( Q^*_1 \) will ascend as \( \varepsilon \) increases if \( a < a' \), but will descend as \( \varepsilon \) increases if \( a \geq a' \), where \( a' = ([((\theta - 1)v - \alpha(m + d))/(2 - \alpha)] \).

**[Proof]** See appendix F.

Furthermore, this paper is interested not only in what factors affect the optimal initial capacity decision in a two-stage scenario, but in how they influence the decision. Hence, the following two propositions are prominent to gain insight into the influences concerned.

**Corollary 3:**

Under \( d < m < a < (\theta - 1)v \), if \( Q_1 \sim UNI(Q_e - \varepsilon, Q_e + \varepsilon) \) and \( Q_2 = Q_1 \), then the optimal initial capacity level in stage one \( Q^*_1 \) will ascend with an increase in \( Q_e \), \( \theta \), or \( v \), but will descend with an increase in \( a \), \( m \), or \( d \), where \( Q_e = (Q + \overline{Q})/2 \).

**[Proof]** See appendix G.

Through corollary 3, it can be seen that an increase in the expected demand for stage one, the penalty factor, or the normal unit variable cost will lead to a rise in the optimal initial
capacity level in stage one. However, an increase in the cost coefficient of setting additional capacity, maintaining capacity, or reducing capacity will have an adverse impact on the optimal capacity.

Corollary 4:

Under \( \bar{Q}_1 \sim UNIF(Q, \bar{Q}) \) and \( Q_2 = Q_1 \), then the optimal initial capacity level in stage one \( Q_{1*} \) will descend with an increase in \( \alpha \) if \( \theta > 1 + \left[ a(m+a)/(dv) \right] \), but will ascend with an increase in \( \alpha \) if \( \theta \leq 1 + \left[ a(m+a)/(dv) \right] \).

[Proof] See appendix H.

On the other hand, in corollary 4, it is shown the impact of the discount factor on the optimal initial capacity is not constant. In other words, a change in the discount factor can have a positive or negative influence on the optimal initial capacity, depending on whether the penalty factor is relatively large enough or not.

**CONCLUSIONS**

The uncertainty of demand for product has been a prominent issue in many management decisions, especially in the initial stage of providing a new product. Whether it is merchandise or a service, the capacity decisions may involve much more uncertain product demands. For example, in the beginning of operations, an owner of a chain store needs to determine the adequate number of chain stores to provide a new product (or service) even when the demand for the new product is quite uncertain. Following a period of uncertain demand, the owner may need to further consider whether to increase or decrease the number of chain stores, provided the demand has become more stable or predictable.

This paper first presents a single stage model to deal with the issue concerning the impact of demand uncertainty on initial capacity decisions, and then describes a two-stage model to consider possible capacity adjustments after product demand becomes stable and certain. In the second model, two possible demand stages are considered, including an uncertain (or unstable) demand period and a stable demand period. Contingent on the characters of the products, in an uncertain demand period, demand variances may be totally different among various products.
Hence, how to determine an optimal initial capacity level is a significant management decision, which affects future operation income.

The results from the two-stage model indicate that the optimal initial capacity in stage one will depend on several factors, including the expected demand in stage one, the cost coefficient of setting additional capacity, the normal unit variable cost, the penalty factor, the possible demand range (or demand uncertainty) in stage one, and the discount factor, as well as the cost coefficients of maintaining and reducing capacity. Furthermore, the optimal initial capacity level will ascend with an increase in the expected demand for stage one, the penalty factor, or the normal unit variable cost, but will descend with an increase in the cost coefficient of setting additional capacity, maintaining capacity, or reducing capacity.

While the impact of demand uncertainty on optimal capacity is dependent on the relationship between the cost coefficient of setting additional capacity above the basic capacity and other variables, including the discount factor, the cost coefficient of maintaining capacity and the cost coefficient of reducing capacity, this paper finds that the optimal initial capacity level in stage one will ascend as demand variance increases if the cost of adding the capacity in stage two is relatively insignificant.
REFERENCES


Appendix A:

Let \( E(TC) \) be the total expected production costs. Thus, we have

\[
E(TC) = \int_{Q_0}^{Q} \left( \frac{1}{2} \epsilon \left( F_0 + a(k-1)Q_0 + \nu q \right) \right) dq + \int_{Q}^{Q_0} \left( \frac{1}{2} \epsilon \left( F_0 + a(k-1)Q_0 + \nu Q_c + \theta(k-q-Q_0) \right) \right) dq
\]

\[
= \int_{Q_0}^{Q} \left( \frac{1}{2} \epsilon \left( F_0 + a(k-1)Q_0 + \nu q \right) \right) dq + \int_{Q}^{Q_0} \left( \frac{1}{2} \epsilon \left( F_0 + a(k-1)Q_0 + \nu Q_0 + \theta(k-kQ_0) \right) \right) dq
\]

\[
= F_0 + a(k-1)Q_0 + \nu k^2Q_0^2/4\epsilon - Q_0^2/4\epsilon - (\theta-1)kQ_0\overline{Q}/2\epsilon + (\theta-1)kQ_0^2\overline{Q}/2\epsilon + \theta\epsilon\overline{Q}^2/4\epsilon - \theta\epsilon k^2 Q_0^2/4\epsilon
\]

For getting the optimal initial capacity, we need the following F.O.C., i.e.,

\[
\frac{\partial E(TC)}{\partial k} = aQ_0^2 + \nu Q_0^2 k^*/2\epsilon - (\theta-1)\nu Q_0\overline{Q}/2\epsilon + (\theta-1)\nu Q_0^2 k^*/\epsilon - \theta\epsilon Q_0^2 k^*/2\epsilon = 0
\]

\[
\Rightarrow 2\alpha + \nu Q_0 k^* - (\theta-1)\nu Q_0 \overline{Q} + 2(\theta-1)\nu Q_0 k^* - \theta\epsilon Q_0 k^* = 0
\]

\[
\Rightarrow (\theta-1)\nu Q_0 k^* = (\theta-1)\nu \overline{Q} - 2\alpha a
\]

\[
\Rightarrow k^* = \left[ \overline{Q} - 2\alpha a/(\theta-1) \right] /\nu Q_0.
\]

Also, since \( \partial^2 E(TC) / \partial k^2 = (\theta-1)\nu Q_0 > 0 \) (\( \because \theta > 1 \)), the S.O.C. also holds. Hence, the optimal initial capacity becomes

\[
Q_c^* = k^* Q_0 = \overline{Q} - 2\alpha a/(\theta-1) \nu. \]

Moreover, since \( Q_c^* = \overline{Q} - 2\alpha a/(\theta-1) \nu = Q + 2\epsilon - 2\alpha a/(\theta-1) \nu = Q + 2\epsilon \left[ 1 - a/(\theta-1) \nu \right] > Q \)

(\( \because \theta > 1 \) and \( a < (\theta-1) \nu \) and \( Q_c^* = \overline{Q} - 2\alpha a/(\theta-1) \nu < \overline{Q} \), \( Q < Q_c^* < \overline{Q} \) holds.

Appendix B:

Using the proof of proposition 1, \( Q_c^* = k^* Q_0 = \overline{Q} - 2\alpha a/(\theta-1) \nu \). To ensure \( k \geq 1 \) and \( Q_c \geq Q_0 \), it needs \( \overline{Q} - 2\alpha a/(\theta-1) \nu \geq Q_0 \) or \( a \leq \left( \overline{Q} - Q_0 \right)/(\theta-1) \nu/2\epsilon \). Hence, under \( Q < Q_0 < \overline{Q} \), if \( a \leq \left( \overline{Q} - Q_0 \right)/(\theta-1) \nu/2\epsilon \) (\( < (\theta-1) \nu \)), the optimal initial capacity remains \( Q_c^* = k^* Q_0 = \overline{Q} - 2\alpha a/(\theta-1) \nu \) (\( \geq Q_0 \)); otherwise, it will become \( Q_c^* = 1 \cdot Q_0 = Q_0 \) (\( \because \partial^2 E(TC) / \partial k^2 = (\theta-1)\nu Q_0 > 0 \)). On the other hand, it is straightforward to prove \( Q < Q_0 \leq Q_c^* < \overline{Q} \) by some basic assumptions.
Appendix C:

Using the proof of proposition 1, \( Q^*_{c} = k^* Q_0 = \bar{Q} - 2a/(\theta - 1) \nu < \bar{Q} \) implies that \( k^* < 1 \) under \( \bar{Q} < \bar{Q} \leq Q_0 \). It contradicts the requirement of \( k^* \geq 1 \). Hence, under the S.O.C., \( \bar{Q} = Q_0 \), the optimal value of \( k \) becomes 1, and the optimal initial capacity is \( Q^*_{c} = 1 \cdot Q_0 = Q_0 \).

Appendix D:

Using the result of proposition 1, \( Q^*_{c} = \bar{Q} - 2a/(\theta - 1) \nu = Q_0 + \nu - 2a/(\theta - 1) \nu \). Hence, we have \( \partial Q^*_{c} / \partial \nu = -2a/(\theta - 1) < 0 \) if \( a < (\theta - 1) \nu / 2 \), and \( \partial Q^*_{c} / \partial \nu \leq 0 \) if \( a \geq (\theta - 1) \nu / 2 \).

Appendix E:

Let \( TC \) be total production costs in both stages one and two, and \( E(TC) \) be the total expected production costs. In the following equation, \( \alpha \) denotes the discount factor, i.e., 1/(1+discount rate). Thus, we have

\[
E(TC) = \int_{Q}^{q_{k,0}} \left( \frac{1}{2} \alpha + \alpha Q_{c,1} - Q_{0} \right) + v_{q} + \nu \left[ Q_{c,1} + a \left( Q_{c,1} - q_{0} \right) + v_{q} \right] dq
\]

\[
+ \int_{Q}^{Q_{c,1}} \left( \frac{1}{2} \alpha + \alpha Q_{c,1} - Q_{0} \right) + v_{Q_{c,1}} + \theta \left( Q_{c,1} \right) + m \left( Q_{c,1} \right) + a \left( Q_{c,1} \right) + v_{Q_{c,1}} \right] dq
\]

\[
= F_0 + \alpha (k_{i} - 1) Q_0 + \alpha \left( Q_{0}^{2} - Q_{0}^{2} / 4 \nu + \nu \left( Q_{0}^{2} k_{i}^{2} - Q_{0}^{2} / 4 \nu + 2 \alpha Q_{0} \right) k_{i}^{2} / 2 \nu \right)
\]

\[
+ \theta Q_{0}^{2} / 4 \nu - \theta Q_{0}^{2} k_{i}^{2} / 4 \nu + \alpha (m - d) Q_{0}^{2} k_{i}^{2} / 2 \nu - \alpha (m - d) Q_{0}^{2} / 4 \nu + \alpha (m - d) Q_{0}^{2} / 2 \nu
\]

\[
- \alpha (m - d) Q_{0} \bar{Q} / 2 \nu + \alpha (m - d) Q_{0} \bar{Q} / 2 \nu
\]

For getting the optimal initial capacity, we need the following F.O.C., i.e.,
\[ \frac{\partial E(TC)}{\partial k_1} = aQ_0 + vQ_0^*k_1^* \varepsilon + (1 - \theta)vQ_0^*k_1^* / 2\varepsilon + (1 - \theta)vQ_0^*k_1^* / \varepsilon - \theta vQ_0^*k_1^* / 2\varepsilon \]

\[ + \alpha(m - d)Q_0^*k_1^* / 2\varepsilon + adQ_0^*k_1^* / \varepsilon - adQ_0^2Q_0 / 2\varepsilon - adQ_0^2k_1^* / 2\varepsilon \]

\[ - \alpha(a - m)Q_0^2 / 2\varepsilon + \alpha(a - m)Q_0^2k_1^* / \varepsilon = 0 \]

\[ \Rightarrow [(\theta - 1)v + \alpha(a - m + d)]Q_0^*k_1^* = \alpha(a - m + d)Q - 2\alpha d\varepsilon - 2a\varepsilon + (\theta - 1)vQ \]

\[ \Rightarrow k_1^* = \left[ Q - 2\varepsilon(a + ad)\left[(\theta - 1)v + \alpha(a - m + d)\right]\right] / Q_0. \]

Also, since \( \frac{\partial^2 E(TC)}{\partial k_1^2} = (\theta - 1)vQ_0^2 / 2\varepsilon + \alpha(a - m + d)Q_0^2 / 2\varepsilon > 0 \) (\( \because \theta > 1 \) and \( a > m \)), the S.O.C. also holds. Hence, the optimal initial capacity becomes \( Q_{c1}^* = k_1^*Q_0 = Q - 2\varepsilon(a + ad)\left[(\theta - 1)v + \alpha(a - m + d)\right]. \)

Moreover, since \( Q_{c1}^* = Q + 2\varepsilon - 2\varepsilon(a + ad)\left[(\theta - 1)v + \alpha(a - m + d)\right] = Q + 2\varepsilon\left[1 - (a + ad)\left[(\theta - 1)v + \alpha(a - m + d)\right]\right] > Q \)

(\( \because (\theta - 1)v > a \) and \( a > m \)) and \( Q_{c1}^* = Q - 2\varepsilon(a + ad)\left[(\theta - 1)v + \alpha(a - m + d)\right] < Q \) (\( \because \theta > 1 \) and \( a > m \)), \( Q < Q_{c1}^* < Q \) holds.

**Appendix F:**

Since \( Q_{c1}^* = Q + \varepsilon - 2\varepsilon(a + ad)\left[(\theta - 1)v + \alpha(a - m + d)\right] \) and \( \frac{\partial Q_{c1}^*}{\partial \varepsilon} = 1 - 2(a + ad)\left[(\theta - 1)v + \alpha(a - m + d)\right] \), we have \( \frac{\partial Q_{c1}^*}{\partial \varepsilon} > 0 \) if \( a < \left[(\theta - 1)v - \alpha(m + d)\right] / (2 - \alpha) = a' \), and \( \frac{\partial Q_{c1}^*}{\partial \varepsilon} \leq 0 \) if \( a \geq a' \).

**Appendix G:**

Under \( d < m < a < (\theta - 1)v \) and \( Q_{c1}^* = Q + \varepsilon - 2\varepsilon(a + ad)\left[(\theta - 1)v + \alpha(a - m + d)\right], \) it is
straightforward that
\[ \frac{\partial Q^*_1}{\partial Q_e} = 1 > 0, \]
\[ \frac{\partial Q^*_1}{\partial \mu} = 2 \varepsilon(a + a \mu)/(\theta - 1)v + \alpha(a - m + d)]^2 > 0, \]
\[ \frac{\partial Q^*_1}{\partial v} = 2 \varepsilon \mu(a + a \mu)/(\theta - 1)v + \alpha(a - m + d)]^2 > 0, \]
\[ \frac{\partial Q^*_1}{\partial a} = -2 \varepsilon[(\theta - 1)v + \alpha(a - m + d) - \alpha(a + a \mu)]/(\theta - 1)v + \alpha(a - m + d)]^2 < 0, \]
\[ \frac{\partial Q^*_1}{\partial m} = -2 \varepsilon \alpha(a + a \mu)/(\theta - 1)v + \alpha(a - m + d)]^2 < 0, \]
and
\[ \frac{\partial Q^*_1}{\partial d} = -2 \varepsilon \alpha[(\theta - 1)v + \alpha(a - m + d) - (a + a \mu)]/(\theta - 1)v + \alpha(a - m + d)]^2 < 0. \]

Appendix H:

Since \[ \frac{\partial Q^*_1}{\partial a} = -2 \varepsilon[(\theta - 1)v - a(a - m + d)]/(\theta - 1)v + \alpha(a - m + d)]^2, \]
we have
\[ \frac{\partial Q^*_1}{\partial a} < 0 \quad \text{if} \quad \theta > 1 + [a(a - m + d)/dv], \]
but \[ \frac{\partial Q^*_1}{\partial a} \geq 0 \quad \text{if} \quad \theta \leq 1 + [a(a - m + d)/dv]. \]
Biographical Sketch

Ruey-Ji Guo is currently a professor at Department of Accounting, Soochow University in Taiwan. He earned his Ph.D. degree in Accounting in National Taiwan University. His recent research interests focus on management accounting, audit decision, and environmental policy. His papers are published in *Economic Modelling, Journal of Social Sciences and Philosophy, Sun Yat-Sen Management Review, Taiwan Academy of Management Journal, Journal of Accounting and Corporate Governance*, etc.
中文摘要

在許多管理決策中，產品需求的不確定性，一直是相當重要的議題。特別是在新產品推出的初期，產能決策尤其要面對更大的需求變異。本研究的主要目的即在探討新產品推出初期，需求不確定性對於起始產能決策所可能造成的影響。在兩階段的決策模式分析下，研究發現若第二階段的產能調升成本相對顯著，則第一階段的最適起始產能水準將隨需求變異增加而上升；否則，其將隨需求變異增加而下降。

關鍵詞：產能規劃、成本管理、需求不確定性

通訊地址：郭瑞基，東吳大學會計學系教授，100 台北市貴陽街一段 56 號。
E-mail address: grj@scu.edu.tw