MINIMUM AVERAGE FRACTION INSPECTED FOR MODIFIED CSP-1 PLAN

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ABSTRACT
This paper presents the problem of minimizing the average fraction inspected (AFI) for Govindaraju and Kandasamy’s (2000) single level continuous sampling plan with the acceptance number (modified CSP-1 plan). A solution procedure is developed to find the parameters \( (i, f) \) that will meet the average outgoing quality limit (AOQL) requirement, while also minimizing the AFI for the modified CSP-1 plan when the process average \( \bar{p} \) (\( > \) AOQL) is known.

Keywords: Type I Continuous Sampling Plan (CSP-1 Plan), Average Outgoing Quality (AOQ), Average Outgoing Quality Limit (AOQL), Average Fraction Inspected (AFI)

INTRODUCTION
Dodge (1943) presented a continuous inspection procedure commonly referred to as type I continuous sampling plan (CSP-1 plan). It involves alternating between 100% inspection and sampling inspection. The procedure of a CSP-1 plan is as follows: Inspect every unit until \( i \) successive units are found free of defects, and then inspect units at a frequency \( f \). When a defective unit is found, the inspector reverts to 100%, and continue the inspection unit \( i \) successive units are found free of defects.

Govindaraju and Balamurali (1998) proposed the tightened single-level continuous sampling plan (TCSP-1 plan) for providing consumer protection against poor process
quality and improving administration of the plan. Govindaraju and Kandasamy (2000) introduced the single level continuous sampling plan with the acceptance number (modified CSP-1 plan). The operating procedure of the modified CSP-1 plan is the same as that of CSP-1 plan, except for the sampling phase, once $c+1$ defective units are found, stop sampling inspection and restart 100% inspection. Govindaraju and Kandasamy (2000, p. 829) pointed out that the advantage of the modified CSP-1 plan is to achieve a reduction in the average fraction inspected (AFI) at good quality levels.

The continuous sampling plan is a rectifying inspection plan and its inspection cost is directly proportional to the AFI. The average outgoing quality limit (AOQL) is widely used in the primary index of the CSP-1 plan and other continuous sampling plans as the performance of inspection. Previous researchers (Ghosh, 1988; Resnikoff, 1960; Chen et al., 2001) have addressed the problems of achieving the minimum AFI for the CSP-1 and TCSP-1 plans. In this paper, we present the problem of minimizing the AFI for a modified CSP-1 plan. A solution procedure is developed to find the parameters $(i, f)$ that will meet the AOQL requirement, while also minimizing the AFI for the modified CSP-1 plan when the process average $\bar{p} (> \text{AOQL})$ is known.

**PREVIOUS ESTIMATION OF AOQ AND AFI**

From Govindaraju and Kandasamy (2000, p. 831), the AOQ and AFI functions for the modified CSP-1 plan is given as

$$AOQ = \frac{(c+1)(1-f)pq^i}{f + (c+1-f)q^i}$$ \hspace{1cm} (1.1)

$$AFI = \frac{f(1+cq^i)}{f + (c+1-f)q^i}$$ \hspace{1cm} (1.2)

Where:
- $i$: the clearance number of the 100% inspection stage
- $f$: the sampling frequency ($n = 1/f$ is the sampling interval) of the sampling inspection stage
- $c$: the acceptance number of the sampling inspection stage
$p$: the incoming fraction defective (it can be estimated by the process average $\bar{p}$ when the process is in control), $0 \leq p \leq 1$

$q$: $1 - p$

$AOQ$: the average outgoing quality

$AFI$: the average fraction inspected.

**DETERMINATION OF THE PARAMETERS ($i, f$) SATISFYING THE SPECIFIED AOQL VALUE**

From Appendix A, we can find the parameters ($i, f$) satisfying the maximum $AOQ$ for a modified CSP-1 plan. That is

$$
\text{max } AOQ = p_L = \frac{(c + 1)(1 - f)(p_i - q_i)}{i(c + 1 - f)}
$$

(1.3)

After re-arrangement, Eq. (3) can be rewritten as

$$
p_i = \frac{ip_L(c + 1 - f) + (c + 1)(1 - f)}{(c + 1)(1 - f)(i + 1)}, c > 0
$$

(1.4)

Where:

$p_i$: the incoming quality when AOQ reaches a maximum at particular values of $i, f$ and $c$

$q_i$: $1 - p_i$

$p_L$: the specified AOQL value.

The value of $p_i$ for given $p_L, c$: $i$, and $f$ can be obtained by solving Eq. (4) with suitable numerical method, such as the conventional Newton method, to any desired degree of accuracy.
METHOD OF MINIMUM AFI FOR MODIFIED CSP-1 PLAN

Eq. (4) can be further rewritten as

$$f = \frac{(c+1)(ip_L + q_i - p_i)}{ip_L - (c+1)(p_i - q_i)}; c > 0$$ (1.5)

Substituting Eq.(5) into Eq. (2), we have

$$AFI = \frac{1+ cq^i}{1+ \left( \frac{q_i - p_i}{ip_L + q_i - p_i} \right) cq^i}; c > 0$$ (1.6)

The next step is to find the parameters $(i^*, f^*)$ that will meet the AOQL requirement, while also minimizing the AFI for the modified CSP-1 plan. The mathematical model is to find $(i^*, f^*)$ that

Minimize

$$AFI = \frac{1+ cq^i}{1+ \left( \frac{q_i - p_i}{ip_L + q_i - p_i} \right) cq^i}$$ (1.7)

$$p_L = \frac{(c+1)(1-f)(p_i - q_i)}{i(c+1-f)}$$ (1.8)

Subject to

- $i \geq 0$, integer
- $c > 0$
- $0 \leq f \leq 1$.

Montgomery (1991, p.654) pointed out that, as a general rule, it is not a good idea to choose values of sampling interval $n$ larger than 200 for a CSP-1 plan because the production against bad quality in a continuous run of production then becomes very poor. Hence, we can use the following solution procedure to find the parameters $(i^*, f^*)$ that satisfy the minimum AFI.

Step 1. Set the values of the parameters $c$ and $p_L$. 

Step 2. Estimate the process average $\bar{p}$.

Step 3. Let $f = 1/2$ and adopt the method in the Appendix A to obtain the $P_L^i$ value with the given $(P_L, i)$.

Step 4. Compute the local minimum AFI of Step 3.

Step 5. Repeat Steps 3 and 4 with $f = 1/3, 1/4, \ldots, 1/200$, etc. Terminate when it is obvious that the global minimum AFI has been found. The optimal solution is $(i^*, f^*)$.

**NUMERICAL EXAMPLE**

Assume that the producer adopts the modified CSP-1 plan with the acceptance number $c = 1$ for the process control. The manufacturing process is in statistical control and the process average $\bar{p} = 0.02$. The specified AOQL value of the product quality, $P_L$, is equal to 0.01. The purpose is to find the parameters $(i, f)$ that will meet the AOQL requirement, while also minimizing the AFI for the modified CSP-1 plan.

By solving the above model (7) and (8), we obtain $i^* = 108$, $f^* = 1/6$, and AFI = 0.4965. Table 1 lists the corresponding parameters $(i^*, f^*)$ for the different modified CSP-1 plans.

According to Ghosh (1988, p. 419), the optimal solution for traditional CSP-1 plan is $i^* = 98$ and $f^* = 0.121333$ with AFI = 0.50. From Table 1, we conclude that both traditional CSP-1 plan and modified CSP-1 plan have the approximate minimum AFI.

**CONCLUSIONS**

Govindaraju and Kandasamy’s (2000) modified CSP-1 plan has provided a means for decreasing the inspection effort at good incoming quality levels when compared to traditional CSP-1 plan. In this paper, we have presented the problem of minimizing the AFI for a modified CSP-1 plan. The conclusion shows that both traditional CSP-1 plan and Modified CSP-1 plan have the approximate minimum AFI.
TABLE 1

The corresponding parameters \((i^*, f^*)\) for the different modified CSP-1 plans
\((p_L = 0.01, p = 0.02)\)

<table>
<thead>
<tr>
<th>c</th>
<th>(i^*)</th>
<th>(f^*)</th>
<th>(AFI)</th>
<th>(a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>98</td>
<td>0.1213</td>
<td>0.5000</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>108</td>
<td>1/6</td>
<td>0.4965</td>
<td>-0.69%</td>
</tr>
<tr>
<td>2</td>
<td>113</td>
<td>1/5</td>
<td>0.4959</td>
<td>-0.82%</td>
</tr>
<tr>
<td>3</td>
<td>127</td>
<td>1/5</td>
<td>0.5001</td>
<td>0.032%</td>
</tr>
<tr>
<td>4</td>
<td>137</td>
<td>1/5</td>
<td>0.4990</td>
<td>-0.19%</td>
</tr>
<tr>
<td>5</td>
<td>127</td>
<td>1/4</td>
<td>0.5001</td>
<td>0.021%</td>
</tr>
<tr>
<td>6</td>
<td>134</td>
<td>1/4</td>
<td>0.4998</td>
<td>-0.032%</td>
</tr>
<tr>
<td>7</td>
<td>140</td>
<td>1/4</td>
<td>0.4991</td>
<td>-0.17%</td>
</tr>
</tbody>
</table>

Note: \(a = [(AFI - 0.5) / 0.5] \cdot 100\%

APPENDIX A

DETERMINATION OF THE PARAMETERS \((i, f)\) SATISFYING THE SPECIFIED AOQL VALUE

According to Govindaraju and Kandasamy (2000, p. 831), the AOQ function for the modified CSP-1 plan is given by

\[
AOQ = \frac{(c + 1)(1 - f)pq^i}{f + (c + 1 - f)q^i} \tag{A1}
\]

Where:
- \(i\): the clearance number of the 100% inspection stage
- \(f\): the sampling frequency \((n = 1/f\) is the sampling interval) of the sampling inspection stage
- \(c\): the acceptance number of the sampling inspection stage
- \(p\): the incoming fraction defective (it can be estimated by the process average \(\bar{p}\) when the process is in control), \(0 < p < 1\)
Differentiating Eq. (A1) with respect to $p$ and equating the result to zero (let $\frac{dAOQ}{dp} = 0$), we obtain

$$[f + q'(c + 1 - f)][q'(c + 1)(1 - f) - piq^{i-1}(c + 1)(1 - f)] + piq^{2i-1}(c + 1).$$

$$1 = -f + c + 1 - f = 0 \quad \text{(A2)}$$

Appending the subscript 1 to all values of $p$, indicating concern only for the value of the incoming fraction defective satisfying Eq. (A1) and yielding a maximum AOQ, results in:

$$[f + q'_1(c + 1 - f)](q_1 - p_1i) + p_1i q'_1(c + 1 - f) = 0 \quad \text{(A3)}$$

Eq. (A3) can be rewritten as:

$$f + q'_1(c + 1 - f) = \frac{p_1i q'_1(c + 1 - f)}{p_1i - q_1} \quad \text{(A4)}$$

Substituting the expression for $f + q'_1(c + 1 - f)$ in Eq. (A4) into Eq. (A1) obtains

$$\max_{0 \leq p \leq 1} AOQ = p_L$$

$$= \frac{p_1i q'_1(c + 1)(1 - f)}{f + q'_1(c + 1 - f)}$$

$$= \frac{[p_1i q'_1(c + 1)(1 - f)](p_1i - q_1)}{p_1i q'_1(c + 1 - f)}$$

$$= \frac{(c + 1)(1 - f)(p_1i - q_1)}{i(c + 1 - f)} \quad \text{(A5)}$$

Where:

$p_1$: the incoming quality when AOQ reaches a maximum at particular values of $i$, $f$, and $c$
\( q_i: 1 - p_i \)

\( p_L: \) the specified AOQL value.

Hence, Eq. (A5) can be rewritten as

\[
p_i = \frac{ip_L(c + 1 - f) + (c + 1)(1 - f)}{(c + 1)(1 - f)(i + 1)} \tag{A6}
\]

The value of \( p_i \) for given \( p_L, c, i, \) and \( f \) can be obtained by solving Eq. (A6) with a suitable numerical method, such as the conventional Newton method, to any desired degree of accuracy.

**REFERENCES**


使平均檢驗率為最小的修正連續抽樣計劃
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摘要
本研究考慮使平均檢驗率為最小的Govindaraju與Kandasamy具允收數的單水準連續抽樣計劃（修正連續抽樣計劃）的問題，文中將提出求解程序來獲得滿足特定平均出廠品質界限要求下，當製程平均數為已知時，能達成檢驗率為最小的修正連續抽樣計劃的參數。

關鍵詞: 第一型連續抽樣計劃, 平均出廠品質, 平均出廠品質界限, 平均檢驗率。