A Linguistic Approach to Constructing Fuzzy Numerical Scales

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ABSTRACT

Fuzzy set theory represents an attempt to construct a conceptual framework for a systematic treatment of fuzziness in linguistic variables that are represented in terms of words or sentences. These linguistic variables are interpreted as fuzzy sets characterized by membership functions. Membership function can capture the human’s quantitative meaning of such variables in order to handle such meaning for any machinery. To capture human’s true meaning of words or sentences, constructing their membership functions is important for the success of applications. A new scaling method, named as fuzzy scale, is proposed. Fuzzy scale, built upon the Hersh & Caramazza’s work, provides a simple and heuristic scaling method to capture the meaning of natural language.

Keywords: Fuzzy Set, Membership Functions, Scaling, Linguistic Meaning.

1. INTRODUCTION

Fuzzy set theory, first introduced by Zadeh in 1965 [13], represents an attempt to construct a conceptual framework for a systematic treatment of fuzziness in linguistic variables in terms of words or sentences. These linguistic variables are interpreted as fuzzy sets characterized by membership functions that capture the human’s quantitative meaning of such variables in order for any machinery to handle such meaning. Thus, to capture human’s true meaning of words or sentences, constructing their membership functions is important for the success of applications.

The membership function of a fuzzy set can be constructed as a form of a particular attem or the sake of simplifying the computation such as simple triangular, trapezoidal, bell shapes, or a quadratic. However, to reach the subjective nature of the fuzzy behavior, instead of mathematical postulates, membership functions should be constructed in a meaningful way.

Norwich & Turksen [9] and Zimmermann & Zysno [20] proposed direct subjective
assessments on grades of membership and then constructed membership functions in the statistical matter. Zadeh [14], Bharathi-Devi & Sarma [2], Hisdal’s TEE model [8] and Giles’s payoff function [5] interpreted the grades of membership with probabilities. It is argued by author that the assessment of grades of membership should satisfy the important conceptual ideas implicit in fuzzy set theory (e.g., fuzzy, not crisp, and possibilistic, not probabilistic). In fact, based on Yeh & Sainfort’s [12] empirical investigation, subjects did not obey the rules of probability and also tended to ignore the existence of probabilities.

In the Hersh & Caramazza’s [6] studies, grade of membership is defined and justified for the semantic structure of fuzzy set theory. In their method, the grades of membership of fuzzy sets are defined by the subjective responses of “yes/no” and their corresponding confidence for the smallness of the sizes of squares.

In this paper, a new scaling method, named as fuzzy scale, is proposed. Fuzzy scale build upon the Hersh & Caramazza’s work provides a simple and heuristic scaling method to capture the quantitative meaning of natural language. Besides, opposed to Likert Scale that is often adopted on questionnaires, fuzzy scale provides a new approach to eliciting and quantifying responses from questionnaires. It can be applied to any verbal variables for any types of questions. The details of constructing and formulating fuzzy scale are presented in the next section. An empirical study is shown in section 3 and followed by a section of discussion.

2. FUZZY SCALE

Human mind is used to expressing things verbally, not numerically, and is capable of handling such verbal statements [11,19]. Fuzzy scale, first, allows subjects to express freely on questions. For example, a subject can answer excellent, so so, not so good, or even it stinks to the question, “how do you feel about this skater’s performance?” after s/he watched a figure skater’s program.

Second, numbers are then designed from 0 to an arbitrary number that is decided by system designers. For each number, subjects are asked to indicate whether they feel that each number is appropriate to describe the expression they just gave. 0 means “the worst” to describe that expression, whereas, the largest number means “the best” to describe that expression. The answers are “yes” if subjects feel that the numbers are appropriate to describe the expression; otherwise, the answers are “no”. Take the largest number of 10 for example, the design of fuzzy scale shows in figure 1.
For each Y or N answer subjects just indicated, subjects are then asked to rate how confident they were in their Y/N decision on a 6-point level. 0 stands for “purely guessing”, 1 “unsure”, 2 “not very confident”, 3 “somewhat confident”, 4 “pretty sure”, and finally 5 means “absolutely certain”. A design is in figure 1.

Finally, after the assessment of Y/N answers and the corresponding confidence levels, the grade of membership is defined as:

\[
\text{Grade of membership} = 0.5 + d\left(\frac{r}{10}\right), \quad (1)
\]

Where \(d\) is the binary applicability decision (+1 = “yes”, -1 = “no”) and \(r\) represents the value of confidence level (0 \(\leq r \leq 5\)). Based on formula (1), a membership function of a verbal expression can be easily constructed by fuzzy scale. For example, the grade of membership for a “no” judgment with a confidence level of 5 is 0.0 and the grade of membership for a “yes” response with a confidence level of 3 is 0.8.

3. An Analytical Study

Fuzzy scale has been adopted in Yeh & Sainfort’s [12] experiment which tested a new decision making model with fuzzy alternatives. A total of 47 undergraduate and graduate students of the Department of Industrial Engineering in the University of Wisconsin-Madison participated this experiment. The instrument employed was a self-administered questionnaire in which contained questions regarding individual’s risk taking perspective about gambles. In the experiment, fuzzy scale was employed for the purpose of calculating membership functions of fuzzy variables that represented the subject’s verbal expression about some uncertain situations. A section of subjects’ opinions about the perceived ease and complexity of fuzzy scale was also conducted in the experiment.

Results

In this paper, only the proportion of the
results regarding fuzzy scale is presented. Both quantitative and qualitative results will be discussed.

I. Quantitative part

24 membership functions elicited from a single subject for 24 different situations with their corresponding fuzzy expression are graphically presented in figure 2. The subjects were not given a set of fuzzy terms to choose from but generated all these linguistic terms. The abscissas represent grades of membership and the ordinates represent scale values. An example of the expression *maybe* is described following:

As in figure 3, the grade of membership of each scale value is calculated by plugging both the Y/N and its corresponding confidence level into equation (1).

Therefore, in this case, the membership function of *maybe* is:

\[
\mu_{\text{maybe}} = 0/0 + 0/1 + 0.1/2 + 0.7/3 + 0.8/4 + 1/5 \text{+} 0.9/6 + 0.7/7 + 0.3/8 + 0/9 + 0/10,
\]

and its graphical presentation is shown in Figure 2 (1).

II. Qualitative part

Based on a 4-level selection from "not at all" to "very much". Most of the subjects' responses for both difficulty and complexity on the overall technique of fuzzy scale are "not at all" and "a little" and so as for difficulties of specifying verbal terms. However, the subjects felt that both checking Y/N and rating confidence level for the Y/N answers are "somewhat" difficult. Most of them preferred describing their actual feeling for the provided situations verbally to using numbers.

Compared to the method of directly assigning crisp numbers, the process of checking Y/Ns and confidence levels is easier. Instead of struggling with the choosing the exact crisp numbers on the conventional numerical scale, most subjects assured all answers on fuzzy scale. Some of the subjects thought that the questions on fuzzy scale were complex due to many different aspects to consider. Quite a few of them, however, thought not because once they knew the meaning of the expressions they just gave, checking Y/Ns and providing the corresponding confidence levels did not require much thought. All subjects felt comfortable on fuzzy scale after practice.
Figure 2  Graphical representation of membership functions for different fuzzy terms
Constructing Fuzzy Numerical Scales

The Y/N answers to describe *maybe* are:

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And the confidence levels for Y/N answers are:

|   | 5 | 5 | 4 | 2 | 3 | 5 | 4 | 2 | 2 | 5 | 5 |

*Figure 3  An example for the design of the *maybe* fuzzy scale*

4. DISCUSSION

We compare fuzzy scale with two types of common scaling methods that are most applied to close questions in questionnaire. The first one is to design scales in words such as from “satisfying” to “not satisfying”, from “excellent” to “poor”, and so on. Since those words represent subjective perspectives, same word “excellent” for example may not be the same meaning for two individuals. Furthermore, for the data analyses, those words are usually quantified by the usage of crisp numerical scales, i.e. 1, 2, 3, etc., and therefore, it results in the following problems:

1) opinion elicited from subjects is still identically determined,

2) the essence of applying linguistic terms is lacking, and

3) it is similar to the crisp numerical scales.

The second method is the crisp numerical scale (i.e. Likert Scale) that is designed by the usage of digital numbers from 1 (or 0) to an arbitrary one. Based on Yeh & Sainfort’s observation [12], subjects usually spent more time on choosing one certain number. Moreover, conventional crisp scales are more sensitive to time span. Take a numerical scale from 0 to 5 for example, a subject chooses 2 for his/her answer and later s/he might think 3 is the answer under the same condition. In this case, it will cause an unreliability problem. Nevertheless, fuzzy scale is asserted to derive more reliable answers from subjects. For instance, a subject’s fuzzy scale is as following:

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Confidence level:

|   | 5 | 0 | 5 | 5 | 0 | 5 |
The grades of membership are $0/1 + 0.5/1 + 1/2 + 1/3 + 0.5/4 + 0/5$.

Obviously, scales 2 and 3 are both appropriate and certain for the subject and therefore, in this case, fuzzy scale can eliminate the problem of unreliability. Consider another fuzzy scale example:

$$
\begin{array}{cccccc}
0 & 1 & 2 & 3 & 4 & 5 \\
Y & Y & Y & Y & Y & Y \\
N & N & N & N & N & N \\
\end{array}
$$

Confidence level:

$$
\begin{array}{cccccc}
5 & 0 & 5 & 5 & 0 & 5 \\
\end{array}
$$

The grades of membership are $0/1 + 0.5/1 + 1/2 + 1/3 + 0.5/4 + 0/5$.

The grades of membership are exactly the same as the previous example. Two different fuzzy scales can result in the same membership function. Hence, fuzzy scale can produce more reliable results.

Except for the reliability, fuzzy scale can lead to more accuracy results. For example, a Likert scale is a scale of 5 degrees, along with a row of expression, as follow:

Very Bad OK Good Very bad good

$$
\begin{array}{cccccc}
1 & 2 & 3 & 4 & 5 \\
\end{array}
$$

Suppose that a subject feels for a proposition is *good* and chooses 4 based on the scale provided to this subject. Assume that another subject feels for the same proposition is *good* as well but ignores the row of expression. According to his/her judging system, 3 might represent his/her *good*. The analytical result would cause a bias problem. On the contrary, *good* might be denoted “yes” at 3 and 4 in fuzzy scale. The verbal terms expressed by subjects can even be the indicators for the analyses. Hence, the bias could be reduced by the usage of fuzzy scale. Although we claim that fuzzy scale is a subjective-oriented measurement system, in fact, it could perform better than the objective measuring scale.

As we can see in figure 2, graph (1) and graph (20) are showing different membership functions but they both represent fuzzy term *maybe* due to subject’s facing different situations. With verbal terms, individual’s opinion can be freely and easily expressed. Since such verbal terms are fuzzy and cannot be exactly defined, by the aid of its membership function, fuzzy scale can elicit real subjective information.

In conclusion, this study extends Hersh and Caramazza’ work on judging the degrees of membership for fuzzy sets. The contribution of this paper is to physically formulize and refine a subjective measuring scale for the application of managerial study. In addition, as discussed above, fuzzy scale provides a wider range of practice than Likert scale does. In other words, the former performs an objective measures like the latter as well as explicates the substantial subjective judgment that the latter cannot.
elucidate. The usage of linguistic variables and their membership functions derived from fuzzy scales can both qualitatively and quantitatively represent individual’s actual perception. Fuzzy scale is not only suitable for the purpose of being able to quantitatively represent and program the meaning of expression in natural language, but also can provide a general method on scaling.

5. CONCLUSION

Fuzzy scale is a heuristic and simply constructing scaling method to elicit subjective grades of membership for human’s linguistic expression. Based on the semantic explanation in [5], the notion of the meaning of a classical expression can be described as to assert that expression if and only if it is confidently thought to be true. It is believed that a fuzzy notion is a general case of the classical one. Thus, to extend this notion to the practical meaning of a fuzzy term, it is reasonable to define a grade of membership as that the decision making on “yes” and “no” to assert or not to assert the given fuzzy terms and the confidence level on that assertion. By this definition, the subjective assertion (or no assertion) along with its confidence level on numerical scales can not only describe the meaning of fuzzy expression but also extend the classical scaling methods.
REFERENCE


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