Forecasting Taiwan’s major stock indices by the Nash nonlinear grey Bernoulli model

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\begin{abstract}
The mathematics of traditional grey model is not only easy to understand but also simple to calculate. But, the linear nature of its original model results in the inability to forecast the drastically changed data of which essence is in nonlinear. For this reason, this study investigates cases using nonlinear grey Bernoulli model (NGBM) to demonstrate its ability in forecasting nonlinear data. The NGBM is a nonlinear differential equation with power $n$. The power $n$ is determined by a simple computer iterative program, which calculates the minimum average relative percentage error of the forecast model. Furthermore, the authors improve NGBM by Nash equilibrium concept. The Nash NGBM (NNGBM) contains two parameters, the power $n$ and the background value $p$, which increase the adjustability of NGBM model. This newly proposed model could enhance the modeling precision furthermore. In order to validate the feasibility of the NNGBM concept, the NNGBM is applied to forecast the monthly Taiwan stock indices for 3rd quarter of 2008. The forecasting results show: (1) the NNGBM actually improve the forecasting precision, (2) the Taiwan’s stock markets tend to be a bear market from July 2007 to September 2008, and the whole investing environments will prevail with collapsing financial prices, pessimism and economic slowdown.
\end{abstract}

1. Introduction

Stock market indices are important investment information for investors. These indices could help investors to understand what the current state of economic situation is. Investors could refer to this information and decide whether it is a proper time to invest or not. In other words, if the investors could foresee the trend of stock index movement, they should gain profits from the soaring stock price or avoid losses from the falling traded price. Therefore, in this research, an improved grey forecasting model will be proposed to predict the stock index. In Taiwan, there are several traded stock indices, including Taiwan Stock Exchange Capitalization Weighted Stock Index, Taiwan Stock Exchange Electronic Sector Index, Taiwan Stock Exchange Finance Sector Index and Morgan Stanley Capital International Inc (MSCI) Taiwan Index.

In order to predict, there are more than 300 prediction methods developed. Generally, they are divided into two categories, which are qualitative and quantitative. Qualitative forecasting methods include the Delphi methods, trend prediction method, the expert system etc. Quantitative forecasting methods include linear multiple regression analysis, exponential smoothing, time series analysis, neural networks, genetic algorithm, and grey forecasting method (Wen, 2004) etc. Among all forecasting methods, Grey theory, first proposed by Deng (Deng, 1989) in 1982, is novel and draws some attention from academic society. Accumulated generating operation (AGO) is one of the most important feature of grey theory and the purpose of AGO is to reduce the randomness of the raw data. Grey forecasting method has widely applied in many research areas, such as finance, seismology, agriculture, and engineering management (Jiang, Yao, Deng, & Ma, 2004; Lee, 1986; Xu & Wen, 1997; Yong, 1995). In the literatures, there are many methods to forecast the general behavior trend of stock prices by macro economic model, time series method and neural network so on. In this research, the previously proposed NGBM (Chen, 2008; Chen, Chen, & Chen, 2008) together with Nash equilibrium concept, which is called NNGBM, is used to forecast stock market indices.

Although the traditional grey forecasting model could achieve satisfactory precision, the motivation to improve has never stopped. Researchers developed various hybrid grey forecasting model, such as Grey–Fuzzy (Wang, 2002), Grey–Taguchi (Yao & Chi, 2004), Grey–Markov (Hsu, 2003), Grey–Fourier (Hsu, 2003), Grey–deseasonalized Data (Tseng, Yu, & Tzeng, 2001) etc. The mathematics becomes more and more complicated which deviates from the original idea of simplicity of Grey theory. For this reason,
the authors conducted a series of researches to modify the original equation. The first part considered improving the linear model to nonlinear one, which is called nonlinear grey Bernoulli model (NGBM) in our previous works (Chen, 2008; Chen et al., 2008). From the results, the forecasting precision is indeed improved because of the introduction of nonlinear adjustable parameter \( n \) in the modified model. Above all, the simplicity of the original model is kept. In this research, the improvement is further achieved by applying Nash equilibrium concept in Economics. Therefore, a novel model with two adjustable parameters \( n \) and \( p \) is proposed and is called Nash nonlinear grey Bernoulli model (NNGBM). In this study, a numerical example is demonstrated to show this novel model is effective and then this model is applied to forecast the Taiwan’s major stock indices developing tendency. The results could provide the investors as reference of future investing plan and the proposed methodology could be also easily used by the investors or researchers to forecast the future changing trend of stock market not only in Taiwan but in worldwide.

2. Methodology

In Grey theory, the accumulated generating operation (AGO) technique is applied to reduce the randomization of the raw data. These processed data become monotonic increase sequence which complies with the solution of first order linear ordinary differential equation. Therefore, the solution curve would fit to the raw data with high precision. In some cases, if the original data hold with high degree of nonlinearity, the precision of traditional grey forecasting model will be lowered than linear cases. In this research, we will adopt our previous proposed modified grey forecasting model NGBM together with the Nash equilibrium concept to further improve the forecasting precision. In the following section, the derivations of GM, NGBM, and Nash NGBM are briefly described:

2.1. Grey model, GM \((1, 1)\)

Step 1: Assuming the original series of raw data contains \( m \) entries:
\[
X(0)(m) = \{x(0)(1), x(0)(2), \ldots, x(0)(k), \ldots, x(0)(m)\},
\]  
(1)

where \( x(0) \) stands for the non-negative original historical time series data.

Step 2: Construct \( X(1) \) by one time accumulated generating operation (1-AGO), which is
\[
X(1)(m) = \{x(1)(1), x(1)(2), \ldots, x(1)(k), \ldots, (m)\},
\]  
(2)

where,
\[
x(1)(k) = \sum_{i=1}^{k} x(0)(i), \quad k = 1, 2, \ldots, m.
\]  
(3)

Step 3: The result of 1-AGO is monotonic increase sequence which is similar to the solution curve of first order linear differential equation. Therefore, the solution curve of following differential equation represents the approximation of 1-AGO data:
\[
\frac{dx^{(1)}}{dt} = ax^{(1)} = b,
\]  
(4)

where \( a \) represents grey predicted value. The \( a \) and \( b \) are model parameters. \( x^{(1)}(1) = x(0)(1) \) is the corresponding initial condition.

Step 4: The model parameters \( a \) and \( b \) can be solved by discretization of Eq. (4):
\[
\frac{dx^{(1)}}{dt} = \lim_{\Delta t \to 0} \frac{x^{(1)}(t + \Delta t) - x^{(1)}(t)}{\Delta t}.
\]  
(5)

If the sampling time interval is unit, then let \( \Delta t \to 1 \), Eq. (5) will be reduced to:
\[
\frac{dx^{(1)}}{dt} \approx x^{(1)}(k + 1) - x^{(1)}(k) = x^{(0)}(k + 1),
\]  
(6)

and the second term of Eq. (4) is approximated by
\[
\dot{x}^{(1)}(t) = px^{(1)}(k) + (1 - p)x^{(1)}(k + 1) = z^{(1)}(k),
\]  
(7)

where \( p \) is called background value and its value is in a close interval \([0, 1] \). Traditionally, \( p \) is set to be 0.5. Substitute Eqs. (6) and (7) into Eq. (4), and the source model can be obtained:
\[
x^{(0)}(k) + az^{(1)}(k) = b, \quad k = 2, 3, 4, \ldots
\]  
(8)

From Eq. (8), by least square method, the model parameters \( a \) and \( b \) can be obtained as
\[
\begin{bmatrix}
a \\
b
\end{bmatrix} = \left( B^T B \right)^{-1} B^T Y_n.
\]  
(9)

where \( B \) and \( Y_n \) are defined as follows:
\[
B = \begin{bmatrix}
-z^{(1)}(2) & 1 \\
-z^{(1)}(3) & 1 \\
\vdots & \vdots \\
-z^{(1)}(m) & 1
\end{bmatrix}, \quad Y_n = \begin{bmatrix}
x^{(0)}(2) \\
x^{(0)}(3) \\
\vdots \\
x^{(0)}(m)
\end{bmatrix}.
\]  
(10)

Step 5: Solve the Eq. (4) together with initial condition, and the particular solution is
\[
\dot{x}^{(1)}(k + 1) = \left(x^{(0)}(1) - \frac{b}{a}\right)e^{-ak} + \frac{b}{a}, \quad k = 2, 3, 4, \ldots
\]  
(11)

Hence, the desired prediction output at \( k \) step can be estimated by inverse accumulated generating operation (IAGO) which is defined as
\[
\dot{x}^{(0)}(k + 1) = \dot{x}^{(1)}(k + 1) - \dot{x}^{(1)}(k), \quad k = 1, 2, 3, \ldots
\]  
(12)

or
\[
\dot{x}^{(0)}(k + 1) = (1 - e^{-a}) \left(x^{(0)}(1) - \frac{b}{a}\right)e^{ak},
\]  
(13)

2.2. Nonlinear grey Bernoulli model, NGBM

The Step 1 and 2 are the same as grey model.

Step 3: Eq. (4) is linear differential equation and the only adjustable variable is background value \( p \). Based on the elementary course in ordinary differential equation, a similar form of differential equation to Eq. (4) is called Bernoulli equation (Zill & Cullen, 2000), which is nonlinear and has the following form:
\[
\frac{dx^{(1)}}{dt} + ax^{(1)} = b[x^{(1)}]^n,
\]  
(14)

where \( n \) belongs to any real number except one. Observe the above equation, when \( n = 0 \), the equation reduces to original grey forecasting model, when \( n = 2 \), the equation reduces to Grey–Verhulst equation (Liu, Dong, & Fang, 2004).

Step 4: A discrete form of Eq. (14) is described as
\[
x^{(0)}(k) + az^{(1)}(k) = b[z^{(1)}(k)]^n, \quad k = 2, 3, 4, \ldots
\]  
(15)

By least square method, the above model parameters \( a \) and \( b \) become:
is set to be 0.5 as tradition; the power parameter is historical event, which is intrinsic property, the only adjustable $X(0)$

\[
\begin{bmatrix}
    \alpha \\
    \beta
\end{bmatrix} = (B^T B)^{-1} B^T Y_N,
\]

where $B$ and $Y_N$ are defined as follows:

\[
B = \begin{bmatrix}
-ze^{(1)}(2) & [z^{(1)}(2)]^n \\
-ze^{(1)}(3) & [z^{(1)}(3)]^n \\
\vdots & \vdots \\
-ze^{(1)}(m) & [z^{(1)}(m)]^n
\end{bmatrix},
Y_N = \begin{bmatrix}
x^{(0)}(2) \\
x^{(0)}(3) \\
\vdots \\
x^{(0)}(m)
\end{bmatrix}.
\]

Step 5: The corresponding particular solution of Eq. (14) is

\[
\hat{x}^{(0)}(k+1) = \left(\left(x^{(0)}(1)^{(1-n)} - \frac{b}{\alpha}\right) e^{-n[1-\beta x(k)]} + \frac{b}{\alpha}\right)^{1/(1-n)},
\]

\[n = 1, \quad k = 1, 2, 3, \ldots \]  

2.3. Nash NGBM, NNGBM

In this section, we employ the Nash equilibrium concept in Economics to further improve NGBM. Consider the following optimal problem:

\[
\text{Min} \ c\left(\text{avg}(n, p|x^{(0)})\right),
\]

where $p \in [0, 1], n \in \mathbb{R}^2$. $X^{(0)}$ represents the original series and is exogenous.

**Definition 2.1.** $(p_n, n_n)$ is a Nash solution of function $c(\text{avg})$ if the following conditions hold:

\[
n_0 = \text{Arg}_{(n)}[c\left(\text{avg}(nX^0, p_0 = 0.5)\right)],
\]

\[
p_1 = \text{Arg}_{(p)}[c\left(\text{avg}(pX^0, n_0)\right)],
\]

\[
\vdots
\]

\[
n_i = \text{Arg}_{(n)}[c\left(\text{avg}(nX^0, p_i)\right)],
\]

\[
p_{i+1} = \text{Arg}_{(p)}[c\left(\text{avg}(pX^0, n_i)\right)],
\]

\[
\vdots
\]

\[
n_m = \text{Arg}_{(n)}[c\left(\text{avg}(nX^0, p_m)\right)],
\]

\[
p_{m+1} = \text{Arg}_{(p)}[c\left(\text{avg}(pX^0, n_m)\right)].
\]

The solution curve of traditional GM, Eq. (11) is dominated by the parameters $a$ and $b$ which are related to the raw data sequence $X^{(0)}(m)$ and background value $p$. As $X^{(0)}(m)$ is the result of natural historical event, which is intrinsic property, the only adjustable parameter is $p$. Chang, Lai, and Yu (2005) demonstrated by choosing optimal $p$ values can improve the model precision. For NGBM, $p$ is set to be 0.5 as tradition; the power $n$ is used to be the adjustable parameter. In authors' previous researches (Chen, 2008; Chen et al., 2008), this modified model has been proven to be effective in improving the model precision. Taking into consideration of Chang's idea and Nash equilibrium concept, a modified model is proposed to further improve the forecasting precision of NGBM that is called Nash NGBM (NNGBM). For NNGBM, background value $p$ and the power $n$ are the adjustable parameters. The numerical example will show it is effective in improving the model precision further.

2.4. Rolling grey model, RGM

The characteristic of RGM is taking the latest information into consideration and discards the oldest one, which will keep original data close to the current varying situation. The manipulation strategy of RGM is firstly based on the first $k_0$ data, generally $k_0 = 4$, i.e. $\{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(4)\}$, to build the GM (1, 1), and the forecast fifth value $x^{(0)}(5)$ is obtained. After the actual fifth value appears, the first value of original sequence is eliminated. The new sequence, $\{x^{(0)}(2), x^{(0)}(3), x^{(0)}(4), x^{(0)}(5)\}$ is then used to forecast the sixth value $x^{(0)}(6)$. This procedure is repeatedly ended the sequel. As the financial index are influenced by the latest factors and the historical data are suitable for describing what happened in the past, the RGM fits this phenomenon and is adopted in this research. The momentum strategy and contrarian strategy use the most recent information to make investment decision (Jegadeesh & Titman, 2001). The analysis procedures are summarized as follows.

Assume the original sequence is

\[
X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(k), \ldots, x^{(0)}(m)\}, \quad m \geq 4.
\]

take the partial of original sequence:

\[
X^{(0)}(i;k) = \{x^{(0)}(i), x^{(0)}(i + 1), x^{(0)}(i + 2), \ldots, x^{(0)}(k)\}, \quad i = 1, 2, \ldots, m - 3,
\]

where $k = i + 3$ is frequently used:

![Fig. 1. The curves of raw data and forecast values corresponding to different candidate models.](image)

<table>
<thead>
<tr>
<th>$k$</th>
<th>Original</th>
<th>GM (1, 1)</th>
<th>$P = 0.5$</th>
<th>NGBM</th>
<th>Nash NGBM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x^{(0)}(k)$</td>
<td>$\hat{x}^{(0)}(k)$</td>
<td>$\epsilon(k)$%</td>
<td>$\hat{x}^{(0)}(k)$</td>
<td>$\epsilon(k)$%</td>
</tr>
<tr>
<td>$k = 1$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$k = 2$</td>
<td>2</td>
<td>1.58</td>
<td>20.76</td>
<td>2.00</td>
<td>-0.13</td>
</tr>
<tr>
<td>$k = 3$</td>
<td>1.5</td>
<td>2.07</td>
<td>-38.31</td>
<td>2.0682</td>
<td>-37.88</td>
</tr>
<tr>
<td>$k = 4$</td>
<td>3</td>
<td>2.71</td>
<td>9.45</td>
<td>2.9138</td>
<td>2.87</td>
</tr>
<tr>
<td></td>
<td>$\epsilon(\text{avg})$%</td>
<td>22.84</td>
<td>13.62</td>
<td>13.05</td>
<td></td>
</tr>
</tbody>
</table>
If \( i = 1 \), \( X^{(i)}(1, 4) = \{x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), x^{(0)}(4)\} \),
\( i = 2 \), \( X^{(i)}(2, 5) = \{x^{(0)}(2), x^{(0)}(3), x^{(0)}(4), x^{(0)}(5)\} \),
\[ i = m - 3 \], \( X^{(i)}(m - 3, m) = \{x^{(0)}(m - 3), x^{(0)}(m - 2), x^{(0)}(m - 1), x^{(0)}(m)\} \).

The sequence (22) is employed to build the RGM model, and the forecast value \( \hat{x}^{(i)}(k + 1) \) is obtained. The modeling process can be summarized as

\[
\hat{x}^{(i)}(k + 1) = IAG \cdot GM \cdot AG \cdot x^{(i)}(k), \quad i = 1, 2, 3, \ldots, m - 3.
\]

### Table 2

The actual and forecast values of stock indices using RGM, RNGBM and RNNGBM.

<table>
<thead>
<tr>
<th>TSE Index</th>
<th>Actual</th>
<th>RGM(1, 1)</th>
<th>RNGBM(1, 1)</th>
<th>Nash RNGBM(1, 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Forecast</td>
<td>RPE</td>
<td>ε(avg)</td>
<td>Forecast</td>
</tr>
<tr>
<td>200801</td>
<td>7521.13</td>
<td>8527.41</td>
<td>13.38</td>
<td>3.96</td>
</tr>
<tr>
<td>200802</td>
<td>8412.76</td>
<td>7349.24</td>
<td>12.64</td>
<td>3.28</td>
</tr>
</tbody>
</table>

\( RPE \) is relative percentage error defined as Eq. (24).
\( \langle ε \rangle_{avg} \) is average relative percentage error (ARPE) as Eq. (25).
2.5. Modeling error analysis

To examine the precision of candidate model, error analysis is necessary to understand the difference between fitted value and actual value and to determine the appropriateness of proposed model. Relative percentage error (RPE) compares the recorded and forecast values to evaluate the precision at specific time step k. RPE is defined as

$$\text{RPE} = \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \times 100\% \quad k = 2, 3, \ldots, m.$$  \hspace{1cm} (24)

where \(x^{(0)}(k)\) is the actual value and \(\hat{x}^{(0)}(k)\) is the forecasted value. The total model precision can be defined by average relative percentage error (ARPE) as follows:

$$\text{ARPE} = \frac{1}{k-m} \sum_{i=2}^{m} |\varepsilon(i)|, \quad i = 2, 3, \ldots, m-3.$$  \hspace{1cm} (25)

Models with small \(\varepsilon(\text{avg})\) values are considered as optimal candidate models.

3. Validation of the NNGBM

Proposition 3.1. The following statements hold, therefore NNGBM is more precise and effective than GM and NGBM:

$$\text{ARPE}(p_0, n_1) \leq \text{ARPE}(p_1, n_1) \leq \text{ARPE}(p_0, 0.5) \leq \text{ARPE}(p_0 = 0.5, n_1).$$  \hspace{1cm} (26)

Here, we restate that \(p_0 = 0.5, n = 0\) in traditional GM (1, 1), \(p_0 = 0.5, n = n_1\) in NGBM, and \(p = p_0, n = n_1\) in NNGBM.

To demonstrate the precision and effectiveness of NNGBM, a numerical example is given as follows. A randomly fluctuating sequence \(X^{(0)} = \{1, 2, 1.5, 3\}\) is given. When GM (1, 1) is applied, the average residual error is 22.84%. By adopting NGBM, the average residual error is reduced to 13.62% by selecting optimal power index \(n = -1.5\). The NNGBM holds the minimum error of 13.05% with \(n = -1.7\) and \(p = 0.54\). The results are listed in Table 1 and Fig. 1 shows NNGBM obtaining the satisfactory results.

4. Forecasting Taiwan’s major stock indices

Having demonstrated the ability of NNGBM to further improve the forecasting precision by a numerical example, this research then apply GM, NGBM and NNGBM to forecast five major stock indices in Taiwan, including: (1) Taiwan Stock Exchange Capitalization Weighted Stock Index; (2) Taiwan 50 Index; (3) Electronic Sector Index; (4) Finance Sector Index; and (5) MSCI Taiwan Index. The data used in this study are taken from the Taiwan Stock Exchange (http://www.twse.com.tw/ch/index.php). The sampling period is from July 2007 to June 2008. The feature of stock market indices is that they reflect the returns to straightforward portfolio strategies. If one wishes to buy each share in the index in proportion to its outstanding market value, the value-weighted index would perfectly track capital gains on the underlying portfolio. Similarly, a price-weighted index tracks the returns on a portfolio comprised of equal share of each firm. Therefore, to forecast the future trend of stock index is important to the investors. Furthermore, the research results could provide the governments enacting future financial and economic policy.

The average residual error including forecasting and modeling for each month using the traditional GM, NGBM and NNGBM are tabulated in Table 2. Table 2 shows that the model errors are significantly reduced by applying both NNNGBM and NGBM. The reason is that NNNGBM and NGBM are nonlinear models. The nonlinear ordinary differential equation can adjust the curvature of the solution curve to best fit the original data by adjusting power \(n\), and the authors conclude that the nonlinear model is superior to the traditional linear grey model, as traditional GM is the special case of NGBM by setting \(n = 0\). The NNNGBM could further increase the precision than NGBM because of the adjustable background value \(p\) by Nash equilibrium concept. By considering the power \(n\) and background value \(p\), power \(n\) determines the curve shape that plays the major role in improving forecasting precision. The background value \(p\) could serve as a fine tuning parameter.

All results show that Taiwan’s major securities market tend to be a bear market in the future three months (July 2008 – September 2009) and will be accompanied by falling stock prices. The research results could provide as a reference to financial regulators and the investor, including hedgers and speculators.

5. Conclusions

The conventional grey model is not only easy to understand but also simple to calculate. To enhance the forecasting precision, various kinds of hybrid grey forecasting models have been continually developed. The traditional grey model incorporates with some heuristic methods are proposed, such as fuzzy, neural, Markov chain, and so on. Thus, higher forecasting precision is obtained, while the complexity of mathematics is also obviously increased. This investigation introduces a NNNGBM with fundamental mathematics, and validates its efficiency in reducing forecasting error. Therefore, we introduce NNNGBM to the field of grey theory. In this research, the NNNGBM is applied to forecast stock market indices of the 3rd quarter, 2008 and the results show that bear market is upcoming. The results might serve as a leading indicator for the security market policy makers and as investment information for all investors.

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References


